EXTRACTION OF COULOMB CRYSTALS WITH LIMITED EMITTANCE GROWTH*

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Abstract

Laser Doppler cooled ion traps can produce stationary bunches of ions with extremely low velocity spread (0.6 m/s RMS) and emittance (10^{-13} m normalised). This corresponds to temperatures of a few milli-Kelvin and allows the ions to settle into a fixed lattice analogous to a solid crystal, but with the Coulomb repulsion balanced by the trapping force, rather than a chemical bond. Extraction of such a bunch into a beamline could provide a new regime of ultra-low emittance beams if the emittance is preserved through the extraction operation. This paper shows that extraction from the ion trap and initial acceleration does not cause drastic growth, thus preserving the ultra-low emittance nature of the bunch. Techniques for compensating coherent 'emittance growth' effects such as nonlinear bunch distortion are also investigated.

INTRODUCTION

Ultra-cold bunches with low emittance can produce very small focal points with high density [1, 2]. Paul type ion traps can also form the basis of quantum computers [3] so even the ground state of position can be achieved with laser cooling. Ion traps for accelerator applications have been constructed at Hiroshima University [4] and Rutherford Appleton Laboratory [5]. The Hiroshima ion trap includes laser cooling and forms Coulomb crystals. Extraction has been studied there [6, 7], concentrating on one-dimensional chains of ions. This paper studies extraction and transport of a three-dimensional Coulomb crystal, which does not appear to have been simulated in the literature before.

TRAP FIELD MODEL

The Paul trap geometry is used here, where longitudinal (z axis) focussing of the ions is provided by DC electrodes, while transverse (x, y) focussing is an RF electrostatic quadrupole, which acts analogously to alternating gradient focussing in accelerators.

For extraction studies, a simple field model is desired that still has a near-harmonic central potential and (unlike the harmonic potential) validity to long distances away from the trap. The 150 m wavelength of the $f = 2$ MHz RF is long enough relative to the trap that magnetic fields may be neglected and a point-source electrostatic solution is used for each timestep:

$$V(\mathbf{x}, t) = \sum_n \frac{U_n}{|\mathbf{x} - \mathbf{e}_n|} = \sum_n \frac{U_n^{\mathrm{DC}} + U_n^{\mathrm{RF}} \cos(2\pi f t)}{|\mathbf{x} - \mathbf{e}_n|}.$$

Here, $\mathbf{e}_n$ are the positions of point 'electrodes' with strength U_n. Six electrodes are used:

$$\mathbf{e}_{\pm x} = (\pm r_{\mathrm{trap}}, 0, 0), \mathbf{e}_{\pm y} = (0, \pm r_{\mathrm{trap}}, 0), \mathbf{e}_{\pm z} = (0, 0, \pm r_{\mathrm{trap}}),$$

where $r_{\mathrm{trap}} = 3$ cm. The Paul trap sets

$$U^{\mathrm{RF}}_{\pm x} = u_{\mathrm{tr}}, \quad U^{\mathrm{RF}}_{\pm y} = -u_{\mathrm{tr}}, \quad U^{\mathrm{DC}}_{\pm x, y} = u_{\mathrm{dc}}, \quad U^{\mathrm{DC}}_{\pm z} = u_{\mathrm{lg}} + u_{\mathrm{dc}},$$

with all other values zero. Values of $u_{\mathrm{lg}} = 0.054$ V.m and $u_{\mathrm{tr}} = 5.6452$ V.m produce a spherical Coulomb crystal of $^{40}Ca^+$ at this frequency. An offset u_{dc} is added to all the electrodes, which gives the whole trap a DC bias $\Delta V(\mathbf{0}) = \frac{6u_{\mathrm{dc}}}{r_{\mathrm{trap}}}$ and affects the bunch acceleration when extracted.

To second order, the potential produced by $\mathbf{e}_{+x}$ near $\mathbf{x} = \mathbf{0}$ is

$$V(\mathbf{x}) \simeq \frac{U_{+x}}{r_{\mathrm{trap}}} + \frac{U_{+x} x}{r^2_{\mathrm{trap}}} + \frac{U_{+x}(x^2 - \frac{1}{2}y^2 - \frac{1}{2}z^2)}{r^3_{\mathrm{trap}}},$$

and similarly for the other electrodes. This allows the potential coefficient of x^2, $k_x = \frac{1}{2}\frac{\partial^2 V}{\partial x^2}$, to be calculated

$$k_x = \frac{U_{+x} + U_{-x} - \frac{1}{2}U_{+y} - \frac{1}{2}U_{-y} - \frac{1}{2}U_{+z} - \frac{1}{2}U_{-z}}{r^3_{\mathrm{trap}}},$$

which is proportional to the focussing strength. Cyclic permutations give the formulae for k_y and k_z. Note that $k_x + k_y + k_z = 0$, which is a consequence of $\nabla \cdot \mathbf{E} = 0$. Assuming $U_{+i} = U_{-i}$ and subtracting $\frac{1}{3}(k_x + k_y + k_z) = 0$ gives

$$\frac{2}{3}k_x - \frac{1}{3}k_y - \frac{1}{3}k_z = \frac{2U_{+x} - U_{+y} - U_{+z}}{r^3_{\mathrm{trap}}},$$

which can be satisfied by setting $U_{\pm i} = r^3_{\mathrm{trap}} k_i / 3$.

Including the dynamical focussing effect of the RF gives equivalent smooth focussing strengths $\bar{k}_i$ that can focus in all three places at once, i.e. $\bar{k}_x + \bar{k}_y + \bar{k}_z > 0$. The $\bar{k}_i$ can be derived from k_i^{DC} and k_i^{RF} via numerical integration or finding eigenvalues of the Mathieu equation [8, 9], but there is no analytic formula.

Extraction Methods

The simplest method is **uncontrolled extraction** where U_{+z} is set to zero for $t \geq t_{\mathrm{extract}}$. The beam will pass through $\mathbf{e}_{+z}$ but not encounter a field singularity.

Balanced extraction is also considered, which sets U_{+z} to zero while simultaneously doubling U_{-z}. This has the advantage of keeping k_z the same before and after t_{extract}.

The potentials for these two extraction methods are compared in Fig. 1.

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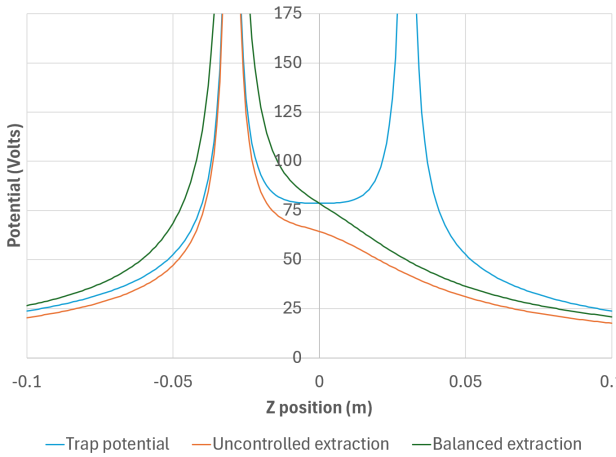

Figure 1: Trap potential along the z axis before and after extraction, for two different methods, with $\Delta V(\mathbf{0}) = 75$ V.

ANALYTIC FORMULA FOR ZERO EMITTANCE GROWTH TRANSPORT

Consider a Coulomb crystal of N ions, with positions $\mathbf{x}_n$ and charge q. The space charge force on particle n is

$$\mathbf{F}_n^{\mathrm{sc}} = \frac{q^2}{4\pi\epsilon_0}\sum_{k\neq n}\frac{\mathbf{x}_n - \mathbf{x}_k}{|\mathbf{x}_n - \mathbf{x}_k|^3}.$$

If the crystal is at equilibrium and zero temperature, then the average force over one RF period should be zero:

$$\mathbf{0} = f\int_0^{1/f}\mathbf{F}_n\,\mathrm{d}t = f\int_0^{1/f}\mathbf{F}_n^{\mathrm{trap}} + \mathbf{F}_n^{\mathrm{sc}}\,\mathrm{d}t = \bar{\mathbf{F}}_n^{\mathrm{trap}} + \bar{\mathbf{F}}_n^{\mathrm{sc}},$$

where $\mathbf{F}_n^{\mathrm{trap}}$ is the force from the trap. Assume that the averaged effect of the DC longitudinal and RF transverse electrostatic focussing is a harmonic potential in each axis:

$$\bar{V}^{\mathrm{trap}} = \bar{k}_x x^2 + \bar{k}_y y^2 + \bar{k}_z z^2 \quad\Rightarrow\quad \bar{E}_i^{\mathrm{trap}} = -2\bar{k}_i x_i$$
$$\Rightarrow\quad \bar{F}_{n,i}^{\mathrm{trap}} = -2q\bar{k}_i x_{n,i}.$$

Combining with the previous result means the time-averaged space charge force is linear in space:

$$\bar{F}_{n,i}^{\mathrm{sc}} = -\bar{F}_{n,i}^{\mathrm{trap}} = 2q\bar{k}_i x_{n,i}.$$

Now consider a distribution of ions that is uniformly scaled by a time dependent factor $\mathbf{x}_n = \alpha(t)\mathbf{x}_n^0$. The space charge force is reduced by the inverse square law $\mathbf{F}_n^{\mathrm{sc}} = \mathbf{F}_n^{\mathrm{sc},0}/\alpha^2$, so if the $\mathbf{x}_n^0$ distribution was the Coulomb crystal at equilibrium,

$$\bar{F}_{n,i}^{\mathrm{sc}} = \frac{\bar{F}_{n,i}^{\mathrm{sc},0}}{\alpha^2} = \frac{2q\bar{k}_i^0 x_{n,i}^0}{\alpha^2}.$$

Here, $\bar{k}_i^0$ are the original focussing strengths used in the trap. Assume that the focussing strengths $\bar{k}_i(t)$ are now time dependent. The total force acting on ion n is

$$\begin{aligned}\bar{F}_{n,i} = \bar{F}_{n,i}^{\mathrm{trap}} + \bar{F}_{n,i}^{\mathrm{sc}} &= -2q\bar{k}_i x_{n,i} + \frac{2q\bar{k}_i^0 x_{n,i}^0}{\alpha^2}\\ &= -2q\bar{k}_i\alpha x_{n,i}^0 + \frac{2q\bar{k}_i^0 x_{n,i}^0}{\alpha^2}.\end{aligned}$$

Using $F = ma$, this should be equal to $m\ddot{x}_{n,i} = m\ddot{\alpha}x_{n,i}^0$. Equating and dividing both sides by $mx_{n,i}^0$ gives

$$\ddot{\alpha} = \frac{2q}{m}\left(-\bar{k}_i\alpha + \frac{\bar{k}_i^0}{\alpha^2}\right) \quad\Rightarrow\quad \bar{k}_i = \frac{\bar{k}_i^0}{\alpha^3} - \frac{m}{2q}\frac{\ddot{\alpha}}{\alpha}.$$

This provides the recipe for choosing $\bar{k}_i(t)$ in order to maintain a given uniform scaling $\alpha(t)$.

SIMULATION

The simulation first prepares a Coulomb crystal of 2000 $^{40}Ca^{+}$ ions in the trap to its equilibrium temperature of a few mK, as shown in Fig. 2. The crystal is spherical as dictated by the chosen electrode strengths.

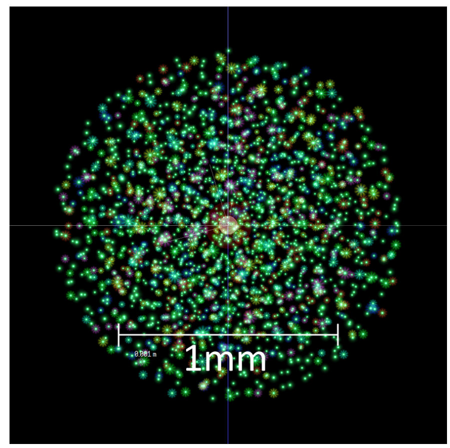

Figure 2: Coulomb crystal of 2000 $^{40}Ca^{+}$ ions in the simulation before extraction.

Once the crystal has settled, either uncontrolled or balanced extraction occurs, as described earlier. Some simulations incorporate **amplitude modulation** (**AM**) of the transverse RF electrode strength to try to satisfy the zero growth transport condition (for the spherical crystal, equal $\bar{k}$ in all three planes). The trap DC bias $\Delta V(\mathbf{0})$ is also varied.

The simulation concludes when the extracted bunch centroid reaches $z = 20$ cm, defining $t = t_{\mathrm{end}}$.

RESULTS

The emittance growth factors in each axis are defined by $G_i = \epsilon_i(t_{\mathrm{end}})/\epsilon_i(t_{\mathrm{extract}})$, where ϵ is the normalised RMS emittance. Fig. 3 shows how the growth varies as a function of DC bias for uncontrolled extraction. Using AM to maintain equal focussing in all planes generally reduces growth, particularly G_z. Note that u_{tr} amplitude was not allowed to exceed the trap value, to prevent large values when the bunch is far away. Uncontrolled extraction has a sudden change in k_z at t_{extract}, leading to a discontinuity in the AM and phase-dependent dynamics, appearing as 'noise' on the graphs as the extraction RF phase is pseudo-random. The lowest 6D emittance growth $G_xG_yG_z$ is obtained at $\Delta V(\mathbf{0}) = 18$ V in both cases, with $G_{x,y,z} = (2.12, 1.88, 11.08)$ for constant u_{tr} and $(1.19, 1.07, 2.39)$ with AM.

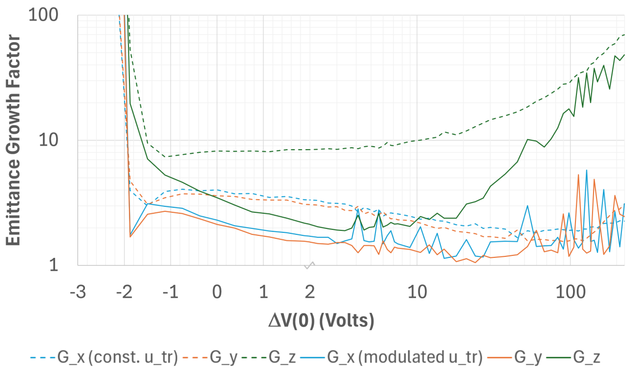

Figure 3: Emittance growth factors for the ‘uncontrolled’ extraction method, with and without AM modulation of u_{tr}.

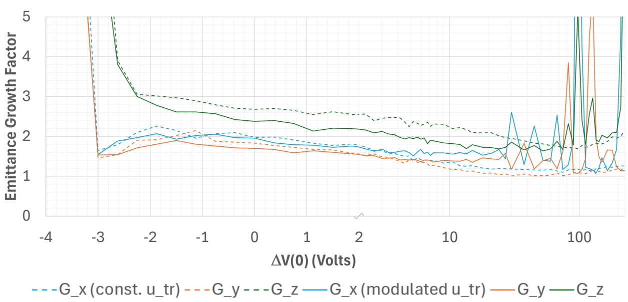

Figure 4: Emittance growth factors for the ‘balanced’ extraction method, with and without AM modulation of u_{tr}.

Figure 4 shows the emittance growth for balanced extraction. This improves greatly over uncontrolled extraction in almost all cases. Adding AM improves G_z but has a marginal effect on transverse growth, sometimes making it worse. The lowest 6D emittance growth is obtained at $\Delta V(\mathbf{0}) = 75$ V in both cases, with $G_{x,y,z} = (1.10, 1.02, 1.74)$ for constant u_{tr} and $(1.17, 1.62, 1.67)$ with AM, which is actually worse than the constant case or the uncontrolled extraction AM case. It seems there is another mechanism for longitudinal growth that is not corrected by AM.

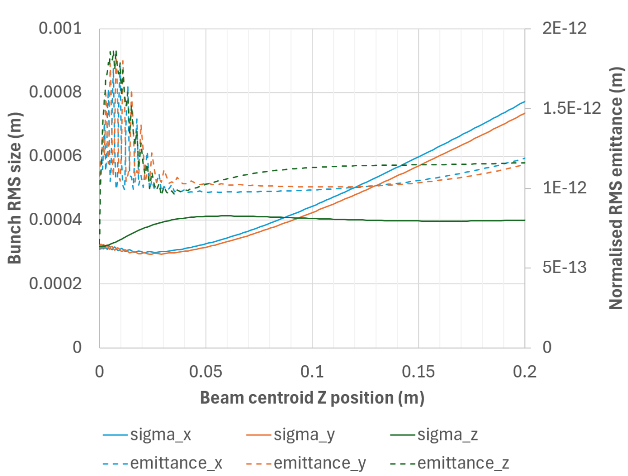

Figure 5: Bunch size and emittance during balanced extraction with optimal DC bias and no u_{tr} modulation.

The bunch size evolution of the best case so far is plotted in Fig. 5 as a function of longitudinal position. Large emittance oscillations are seen near the trap from the nonlinear RF field, which also slightly affects the transverse bunch sizes.

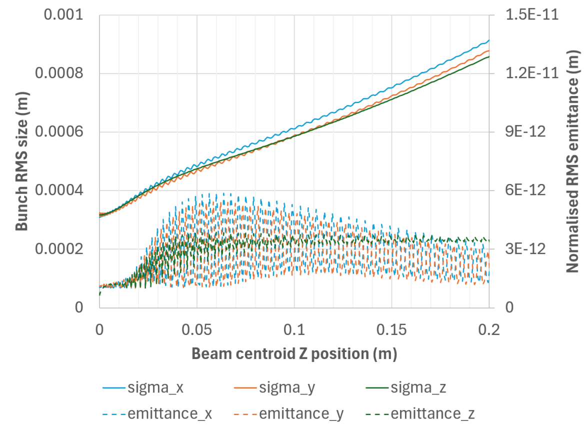

Figure 6: Bunch size and emittance during balanced extraction with unlimited u_{tr} modulation and $\Delta V(\mathbf{0}) = 15$ V.

An example of successful AM is shown in Fig. 6, where the bunch remains near-spherical. Maintaining this required removing the u_{tr} limit, so the RF becomes very strong to affect the bunch far from the electrodes. The emittance continues to be affected by the RF with significant growth. A lower DC bias is used here because higher values make k_z negative at some locations in the potential leaving the trap. This cannot be compensated with AM, since equal alternating gradients always focus.

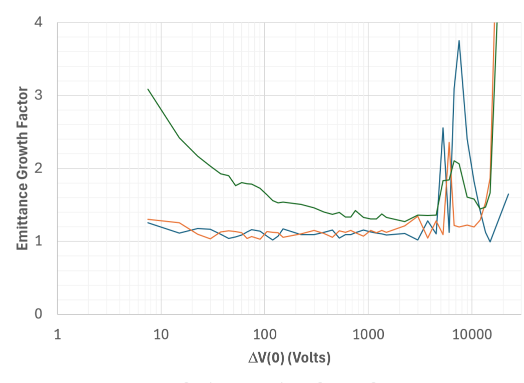

Figure 7: Emittance growth factors with balanced extraction, u_{lg} set to $0.00072\,\Delta V(\mathbf{0})$ and u_{tr} set to maintain a spherical Coulomb crystal (no u_{tr} modulation).

If the ratio between $\Delta V(\mathbf{0})$ and u_{lg} is maintained, then increasing them both will extract more quickly through the same shaped non-linear longitudinal field. Figure 7 shows the emittance growth obtained this way over a wide range of voltages. For the highest voltages, the phase advance in the ion trap becomes high (at the fixed 2 MHz frequency), leading to badly matched transverse dynamics. The lowest 6D emittance growth in this paper (63%) is obtained here at $\Delta V(\mathbf{0}) = 1500$ V, with $G_{x,y,z} = (1.09, 1.12, 1.33)$.

REFERENCES

[1] S. Brooks, "Ultra-Low Emittance Bunches from Laser Cooled Ion Traps for Intense Focal Points", Proc. HB2023. doi:10.18429/JACoW-HB2023-TUC3I1

[2] S. Brooks, "Potential and Issues for Future Accelerators and Ultimate Colliders", Proc. IPAC 2018. doi:10.18429/JACoW-IPAC2018-TUXGBD1

[3] A. Steane, "The ion trap quantum information processor", Appl. Phys. B **64**, 623–642 (1997).

[4] R. Takai, H. Enokizono, K. Ito, Y. Mizuno, K. Okabe and H. Okamoto, "Development of a Compact Plasma Trap for Experimental Beam Physics", Japan. J. Appl. Phys. **45**, No. 6A pp.5332–5343 (2006).

[5] S. L. Sheehy, E. J. Carr, L. K. Martin, K. Budzik, D. J. Kelliher, S. Machida and C. R. Prior, "Commissioning and First Results of the IBEX Linear Paul Trap", Proc. IPAC 2017, pp.4481–4484 (2017).

[6] K. Izawa, K. Ito, H. Higaki and H. Okamoto, "Controlled Extraction of Ultracold Ions from a Linear Paul Trap for Nanobeam Production", J. Phys. Soc. Japan Vol. 79, No. 12, p.124502 (2010).

[7] K. Muroo, H. Okamoto, N. Miyawaki and Y. Yuri, "Simulation study of ultrahigh-precision single-ion extraction from a linear Paul trap", Prog. Theor. Exp. Phys. **2023** 063G01 doi:10.1093/ptep/ptad071

[8] S. Brooks, "Effective Strength of Sinusoidally-Varying Focussing", note available from https://stephenbrooks.org/ap/report/2024-4/effectivek.pdf (2024).

[9] R. Rand with I. Kovacic and S.M. Sah, "Mathieu's Equation and Its Generalizations: Overview of Stability Charts and Their Features", Applied Mechanics Reviews vol. **70**, p.020802 (2018).