Extraction of Coulomb Crystals with Limited Emittance Growth

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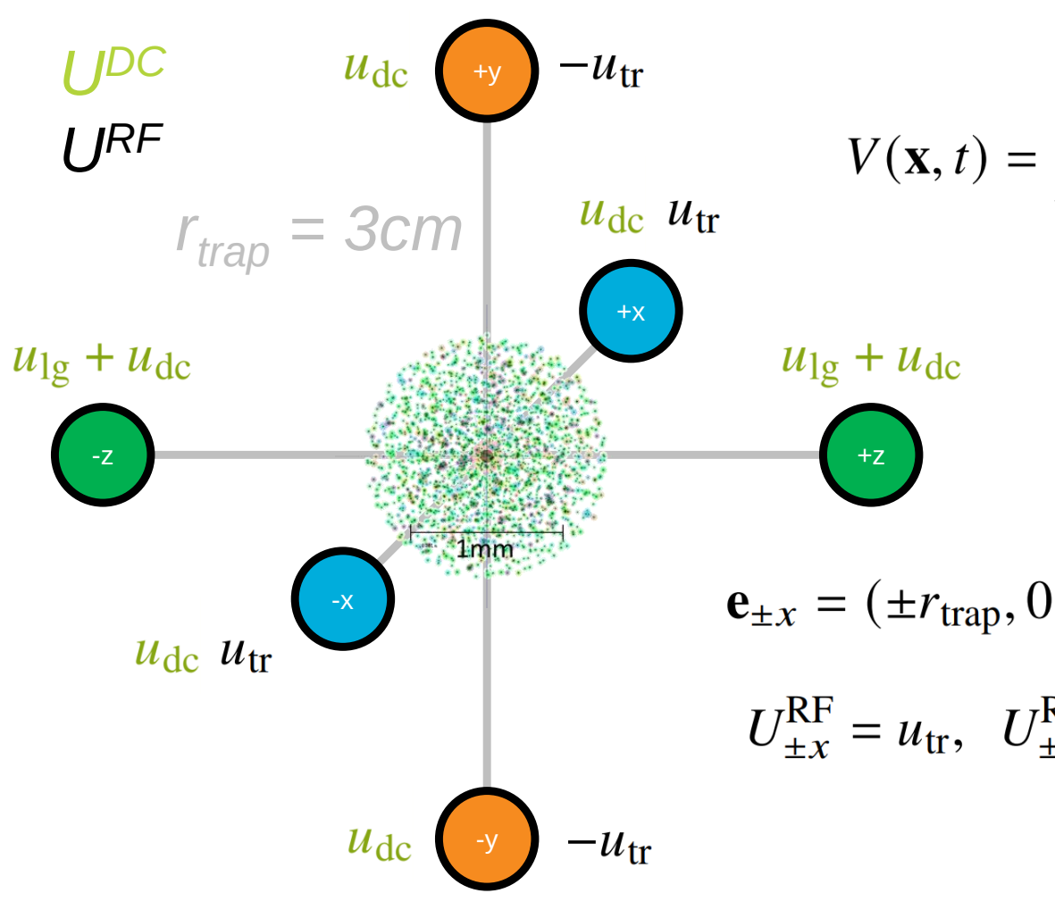
Abstract

Laser Doppler cooled ion traps can produce stationary bunches of ions with extremely low velocity spread (0.6m/s RMS) and emittance (10^{-13}m normalised). This corresponds to temperatures of a few milli-Kelvin and allows the ions to settle into a fixed lattice analogous to a solid crystal, but with the Coulomb repulsion balanced by the trapping force, rather than a chemical bond. Extraction of such a bunch into a beamline could provide a new regime of ultra-low emittance beams if the emittance is preserved through the extraction operation. This paper shows that extraction from the ion trap and initial acceleration does not cause drastic growth, thus preserving the ultra-low emittance nature of the bunch. Techniques for compensating coherent `emittance growth' effects such as nonlinear bunch distortion are also investigated.

Paul Trap Field Model

Simple six point electrode model, f=2MHz.



$$V(\mathbf{x},t) = \sum_n \frac{U_n}{|\mathbf{x}-\mathbf{e}_n|} = \sum_n \frac{U_n^{\mathrm{DC}} + U_n^{\mathrm{RF}}\cos(2\pi f t)}{|\mathbf{x}-\mathbf{e}_n|}$$

$u_{lg}+u_{dc}$ +z

DC bias $\Delta V(\mathbf{0}) = 6u_{dc}/r_{trap}$

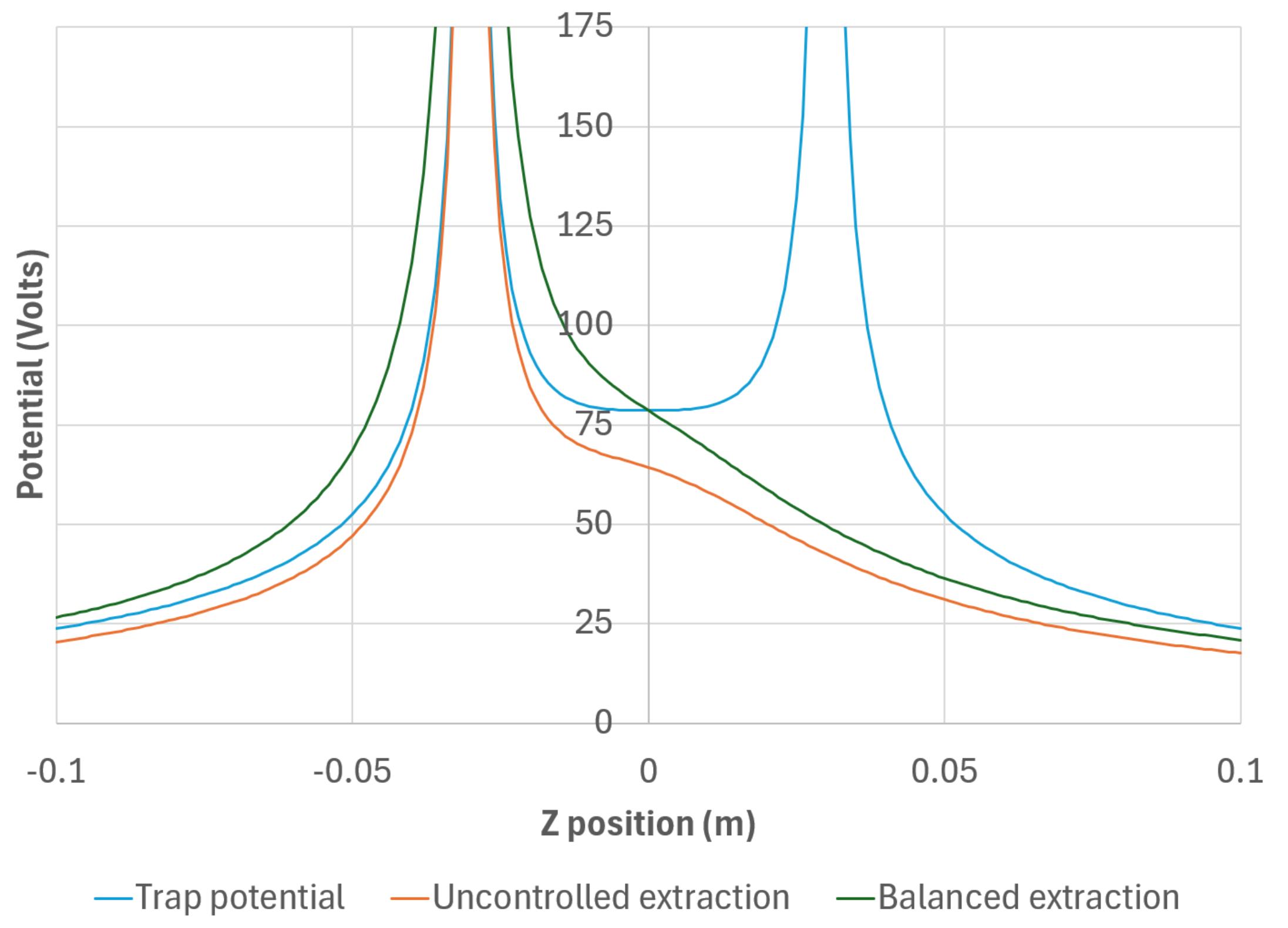
$$\mathbf{e}_{\pm x} = (\pm r_{\mathrm{trap}},0,0), \mathbf{e}_{\pm y} = (0,\pm r_{\mathrm{trap}},0), \mathbf{e}_{\pm z} = (0,0,\pm r_{\mathrm{trap}})$$

$$U_{\pm x}^{\mathrm{RF}} = u_{\mathrm{tr}},\quad U_{\pm y}^{\mathrm{RF}} = -u_{\mathrm{tr}},\quad U_{\pm x,y}^{\mathrm{DC}} = u_{\mathrm{dc}},\quad U_{\pm z}^{\mathrm{DC}} = u_{\mathrm{lg}} + u_{\mathrm{dc}}$$

Extraction Methods

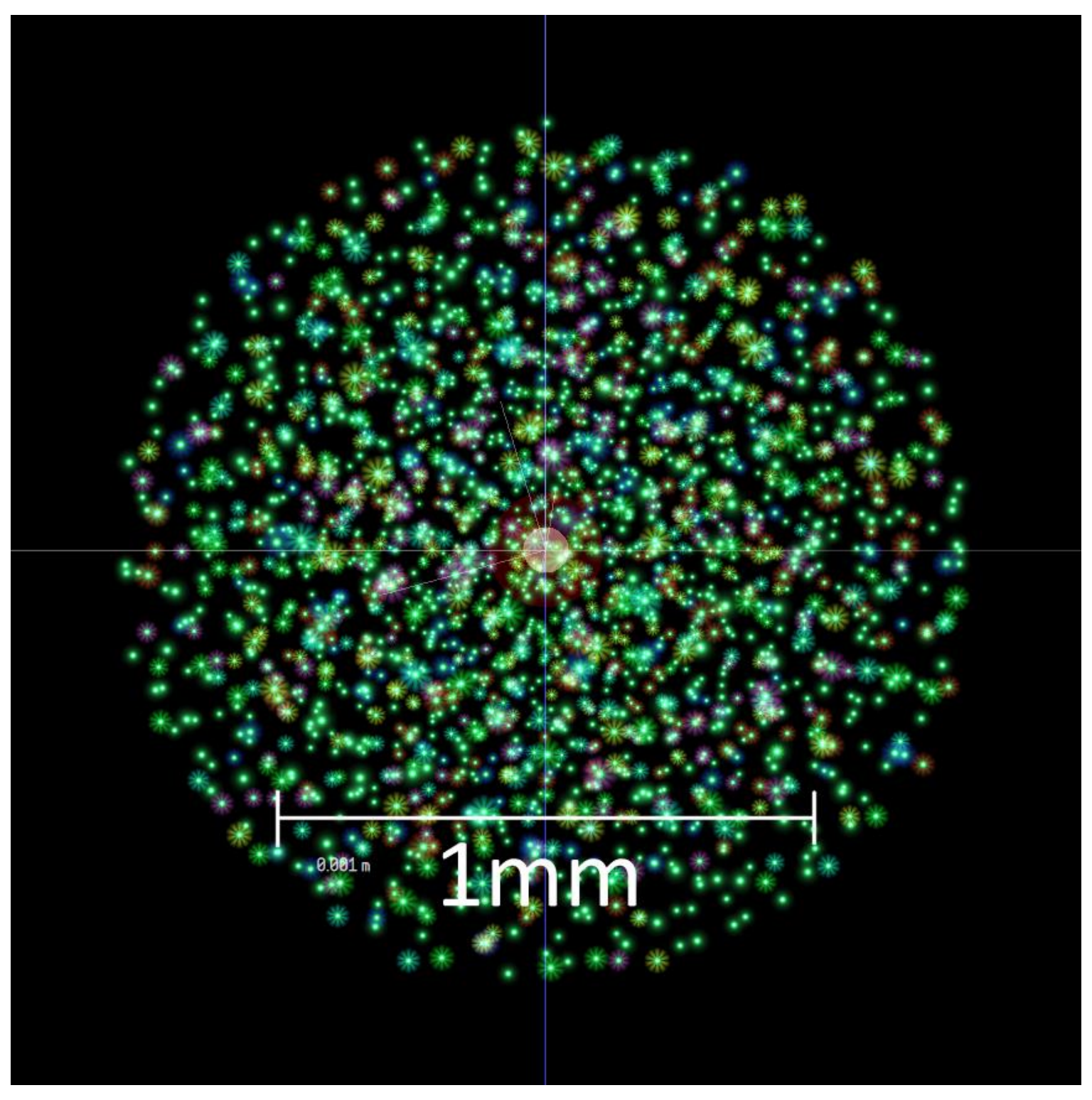
The simplest method is **uncontrolled extraction** where U_{+z} is set to zero for $t \geq t_{extract}$. The beam will pass through $\mathbf{e}_{+z}$ but not encounter a field singularity.

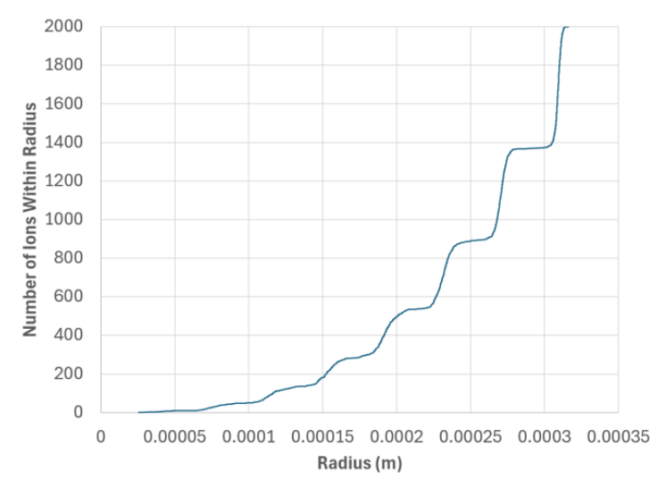
Balanced extraction is also considered, which sets U_{+z} to zero while simultaneously doubling U_{-z}. This has the advantage of keeping d^2V/dz^2 the same before and after $t_{extract}$.



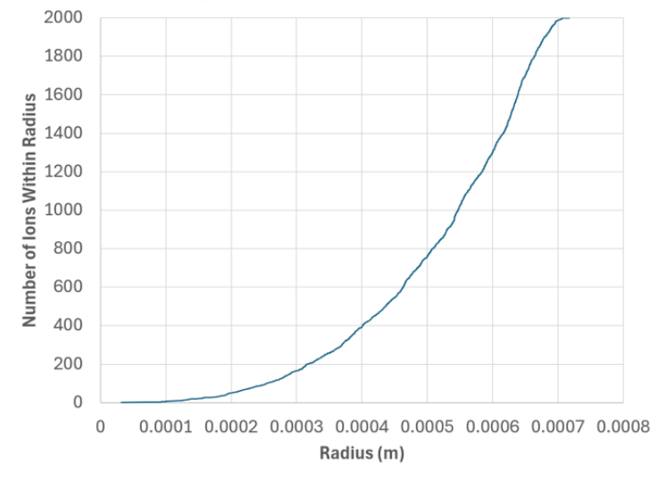
Trap potential along the z axis before and after extraction, for two different methods.

Coulomb Crystal





Shell structure seen in f = 10MHz, T = 1.6mK crystal (above), but washed out in f = 2MHz, T = 4.7mK crystal used here (below).



Coulomb crystal of 2000 $^{40}Ca^+$ ions in the simulation before extraction.

Analytic Formula for Zero Emittance Growth

$$\mathbf{F}_n^{\mathrm{sc}} = \frac{q^2}{4\pi\epsilon_0}\sum_{k\neq n}\frac{\mathbf{x}_n-\mathbf{x}_k}{|\mathbf{x}_n-\mathbf{x}_k|^3}$$

Crystal at equilibrium with its own space charge over an RF cycle

$$\mathbf{0} = f\int_0^{1/f}\mathbf{F}_n\,\mathrm{d}t = f\int_0^{1/f}\mathbf{F}_n^{\mathrm{trap}} + \mathbf{F}_n^{\mathrm{sc}}\,\mathrm{d}t = \bar{\mathbf{F}}_n^{\mathrm{trap}} + \bar{\mathbf{F}}_n^{\mathrm{sc}}$$

Effective linear external focussing force

$$\bar{V}^{\mathrm{trap}} = \bar{k}_x x^2 + \bar{k}_y y^2 + \bar{k}_z z^2 \quad\Rightarrow\quad \bar{E}_i^{\mathrm{trap}} = -2\bar{k}_i x_i$$

$$\Rightarrow\quad \bar{F}_{n,i}^{\mathrm{trap}} = -2q\bar{k}_i x_{n,i}$$

→ Space charge force is also linear

$$\bar{F}_{n,i}^{\mathrm{sc}} = -\bar{F}_{n,i}^{\mathrm{trap}} = 2q\bar{k}_i x_{n,i}$$

Assume extracted bunch scales equally in all axes

$$\mathbf{x}_n = \alpha(t)\mathbf{x}_n^0$$

$$\mathbf{F}_n^{\mathrm{sc}} = \mathbf{F}_n^{\mathrm{sc},0}/\alpha^2$$

Inverse square law

$$\bar{F}_{n,i}^{\mathrm{sc}} = \frac{\bar{F}_{n,i}^{\mathrm{sc},0}}{\alpha^2} = \frac{2q\bar{k}_i^0 x_{n,i}^0}{\alpha^2}$$

x^0 was original crystal, linear force

$$\bar{F}_{n,i} = \bar{F}_{n,i}^{\mathrm{trap}} + \bar{F}_{n,i}^{\mathrm{sc}} = -2q\bar{k}_i x_{n,i} + \frac{2q\bar{k}_i^0 x_{n,i}^0}{\alpha^2}$$

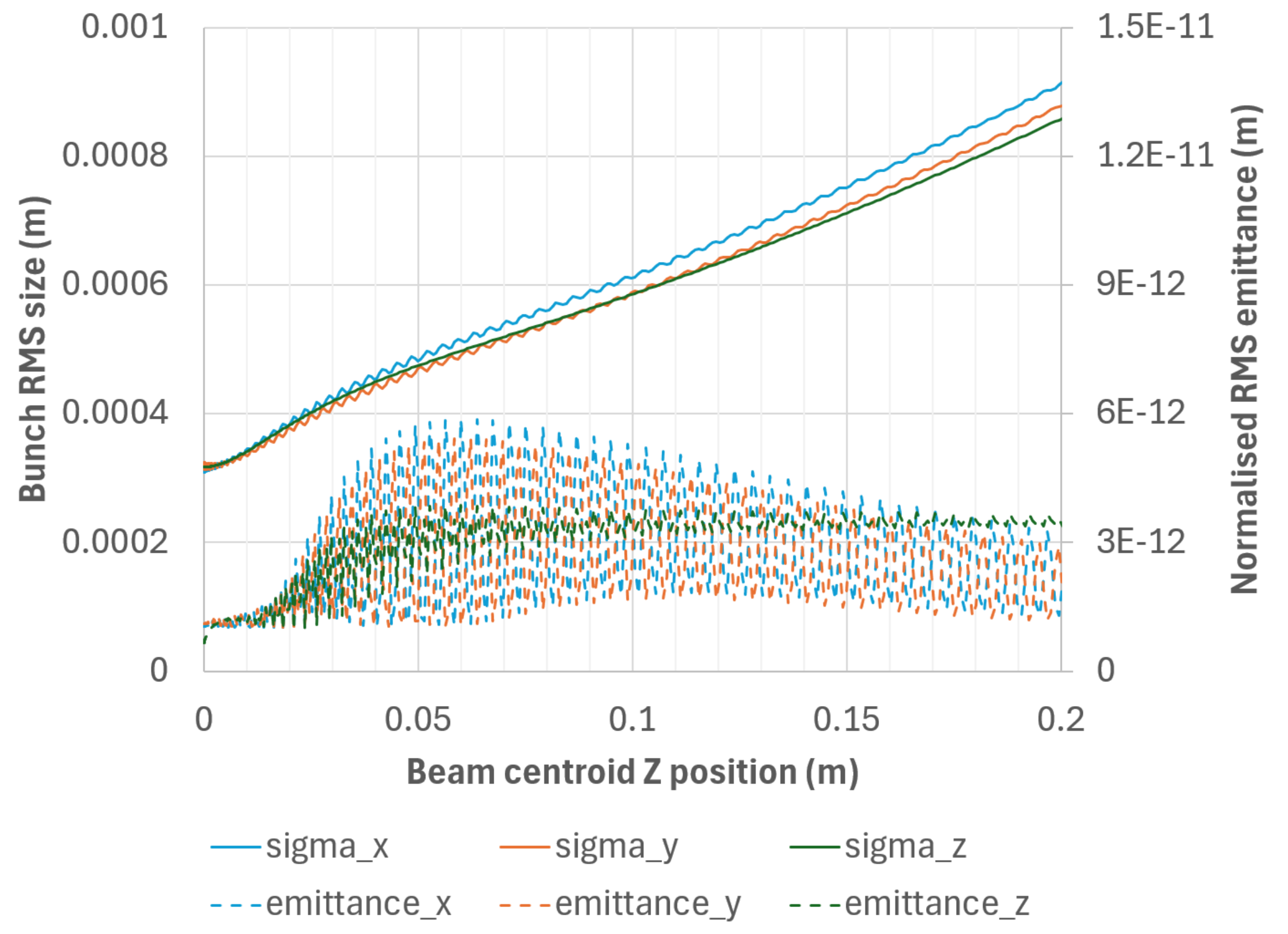
$$m\ddot{x}_{n,i} = m\ddot{\alpha}x_{n,i}^0 = -2q\bar{k}_i\alpha x_{n,i}^0 + \frac{2q\bar{k}_i^0 x_{n,i}^0}{\alpha^2}$$

F=ma

Choose external focussing force to maintain uniform scaling

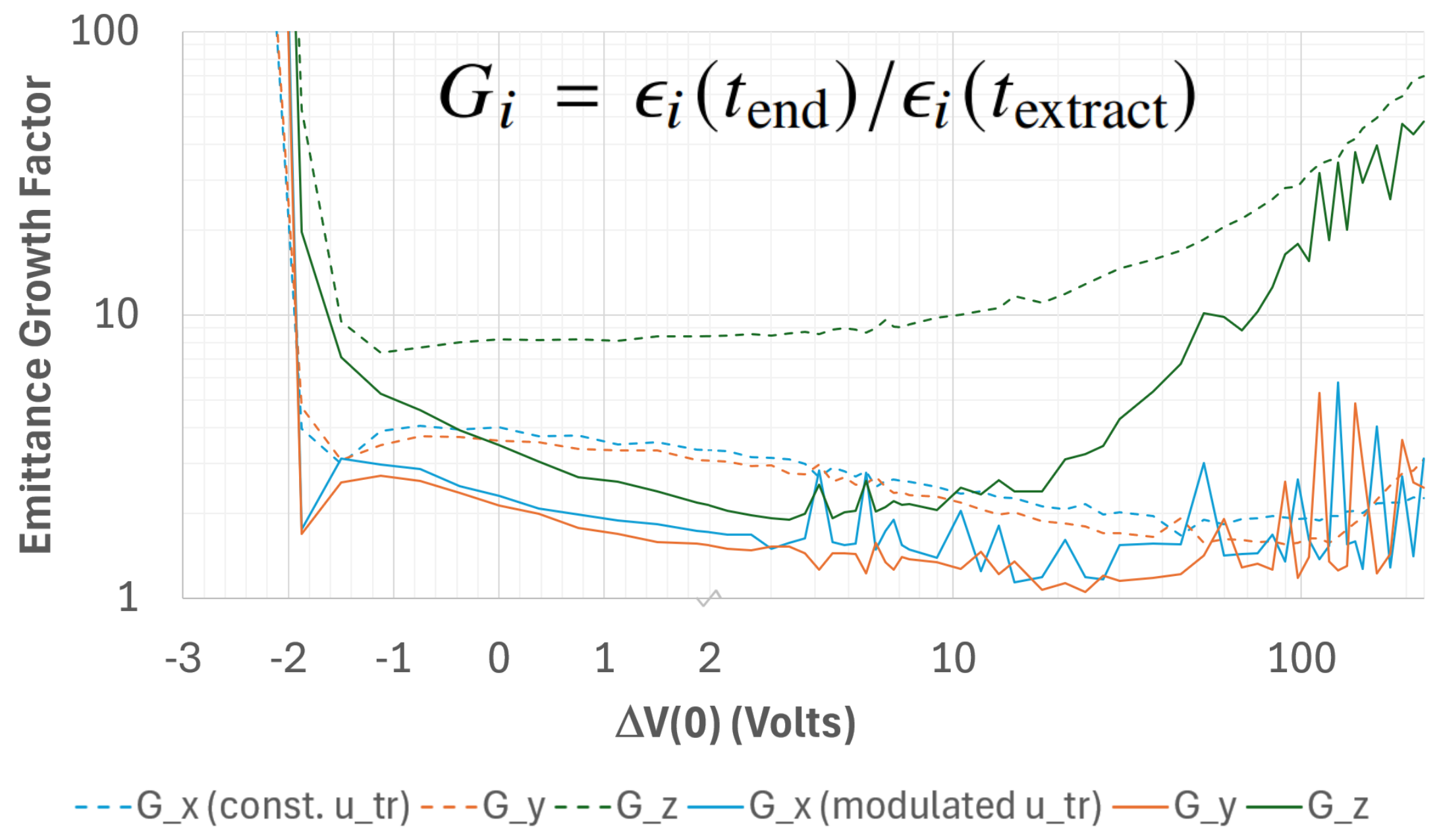
$$\ddot{\alpha} = \frac{2q}{m}\left(-\bar{k}_i\alpha + \frac{\bar{k}_i^0}{\alpha^2}\right) \quad\Rightarrow\quad \bar{k}_i = \frac{\bar{k}_i^0}{\alpha^3} - \frac{m}{2q}\frac{\ddot{\alpha}}{\alpha}$$

Can be done with amplitude modulation (AM) of RF for the transverse trap electrode strength u_{tr}



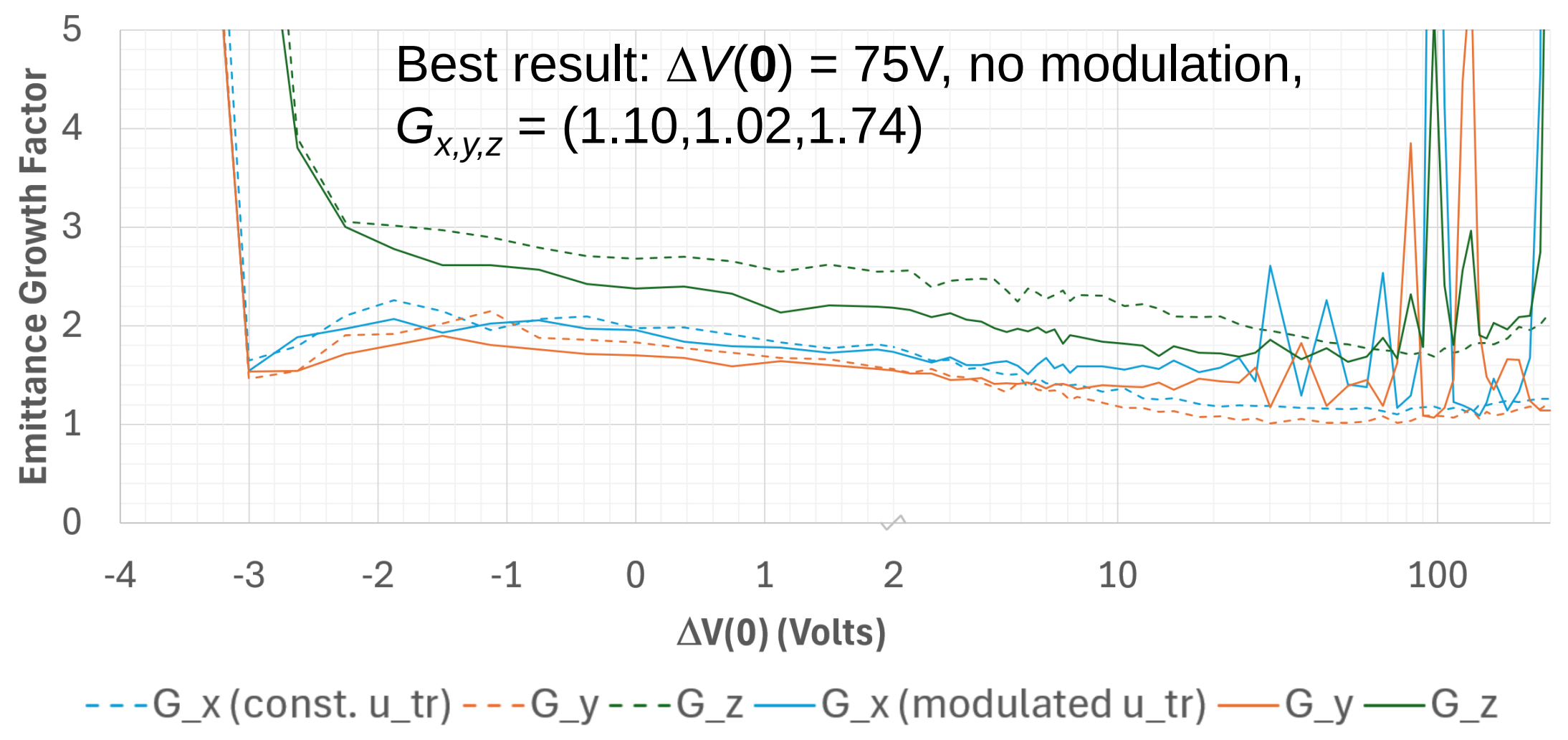
Bunch size and emittance during balanced extraction with unlimited u_{tr} AM modulation and $\Delta V(\mathbf{0})$=15V.

Uncontrolled Extraction

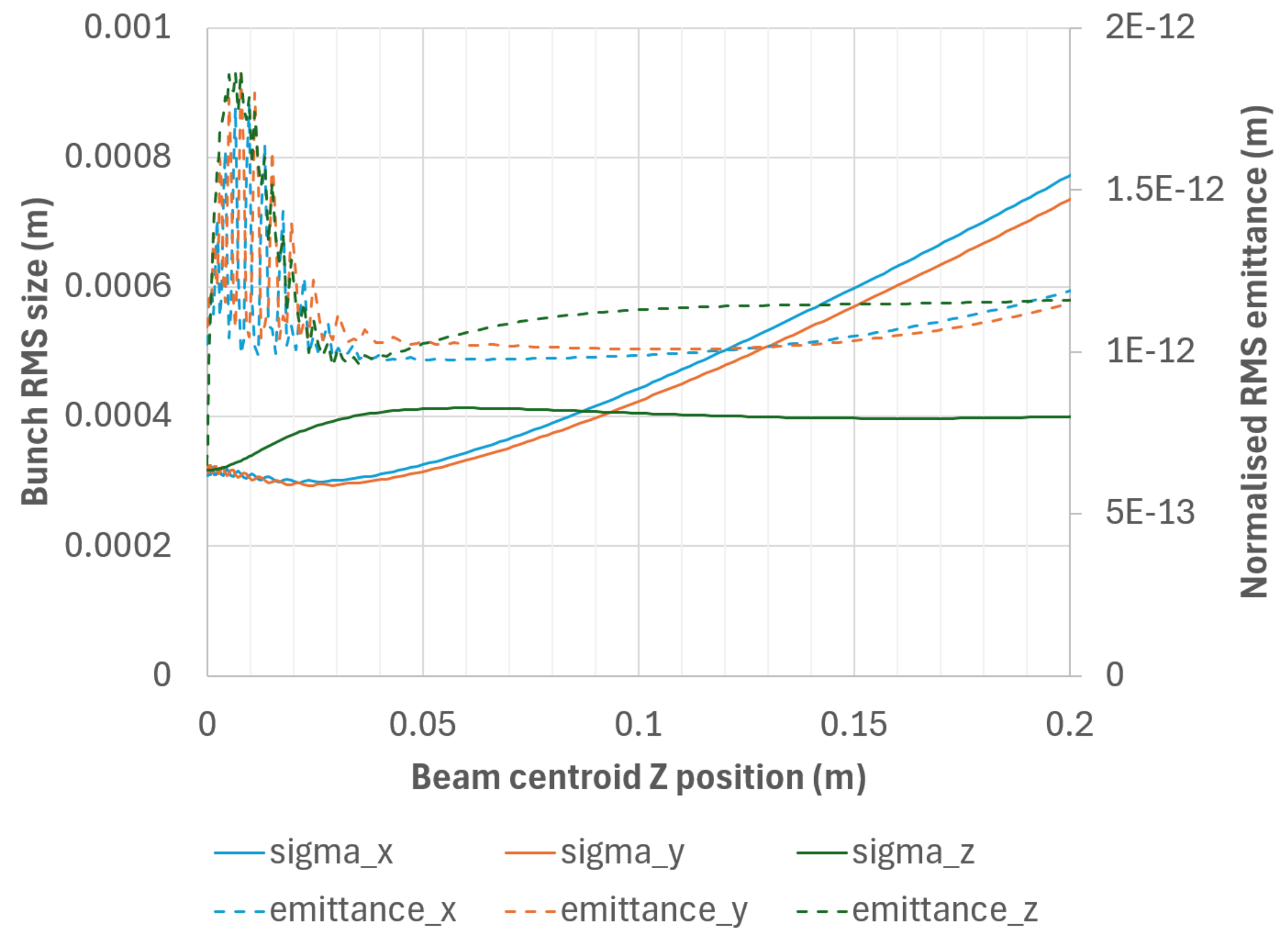


Emittance growth factors, with and without AM modulation of transverse RF focussing.

Balanced Extraction

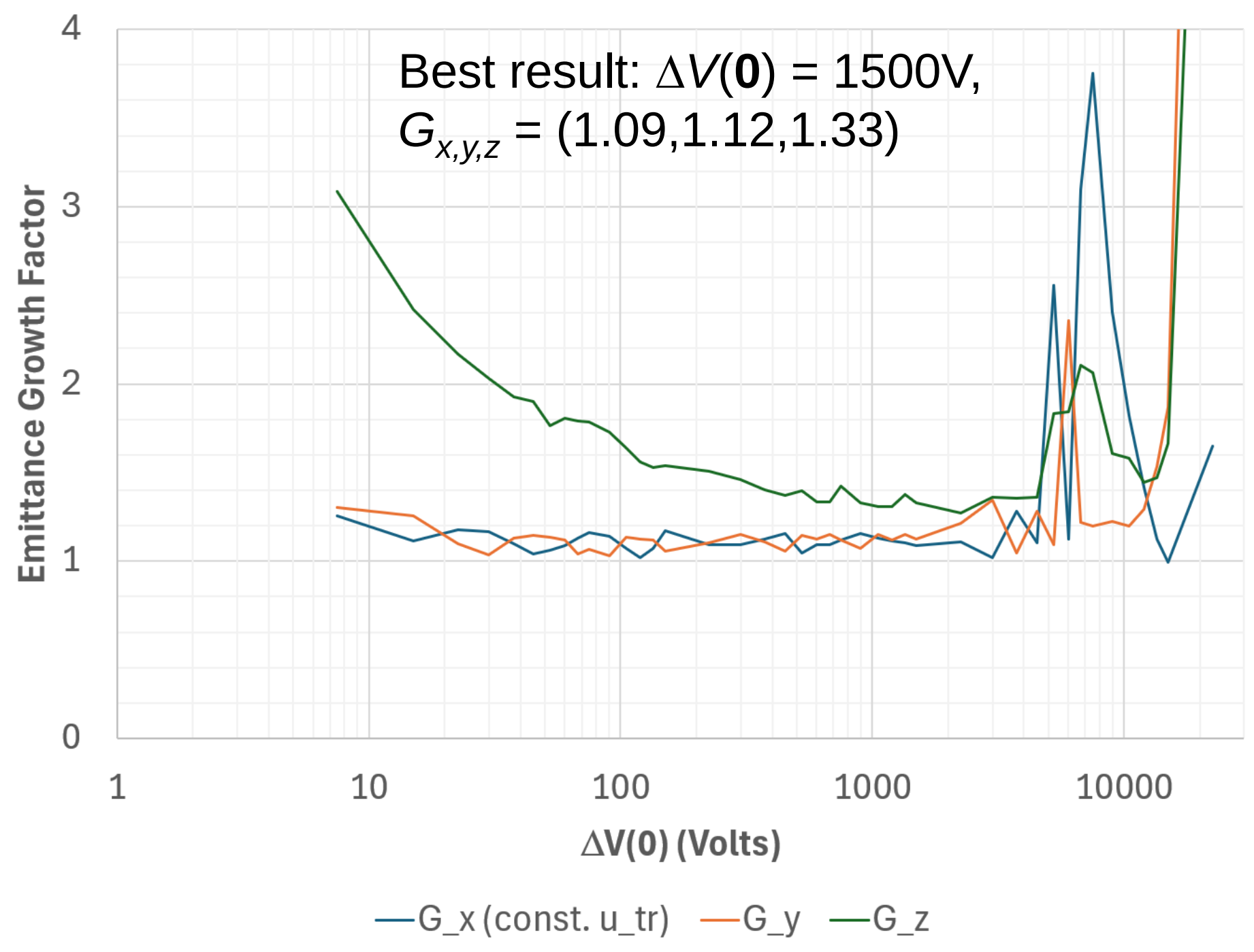

Best result: $\Delta V(\mathbf{0})$ = 75V, no modulation, $G_{x,y,z}$ = (1.10,1.02,1.74)

Improved emittance growth factors, with and without AM modulation.



Bunch size and emittance during balanced extraction with optimal DC bias and no AM.

Increasing Voltage


Best result: $\Delta V(\mathbf{0})$ = 1500V, $G_{x,y,z}$ = (1.09,1.12,1.33)

Varying bias voltage with balanced extraction, u_{lg} set to 0.00072$\Delta V(\mathbf{0})$ and u_{tr} set to maintain a spherical Coulomb crystal (no modulation).

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