

# Scaling VFFAG eRHIC Design

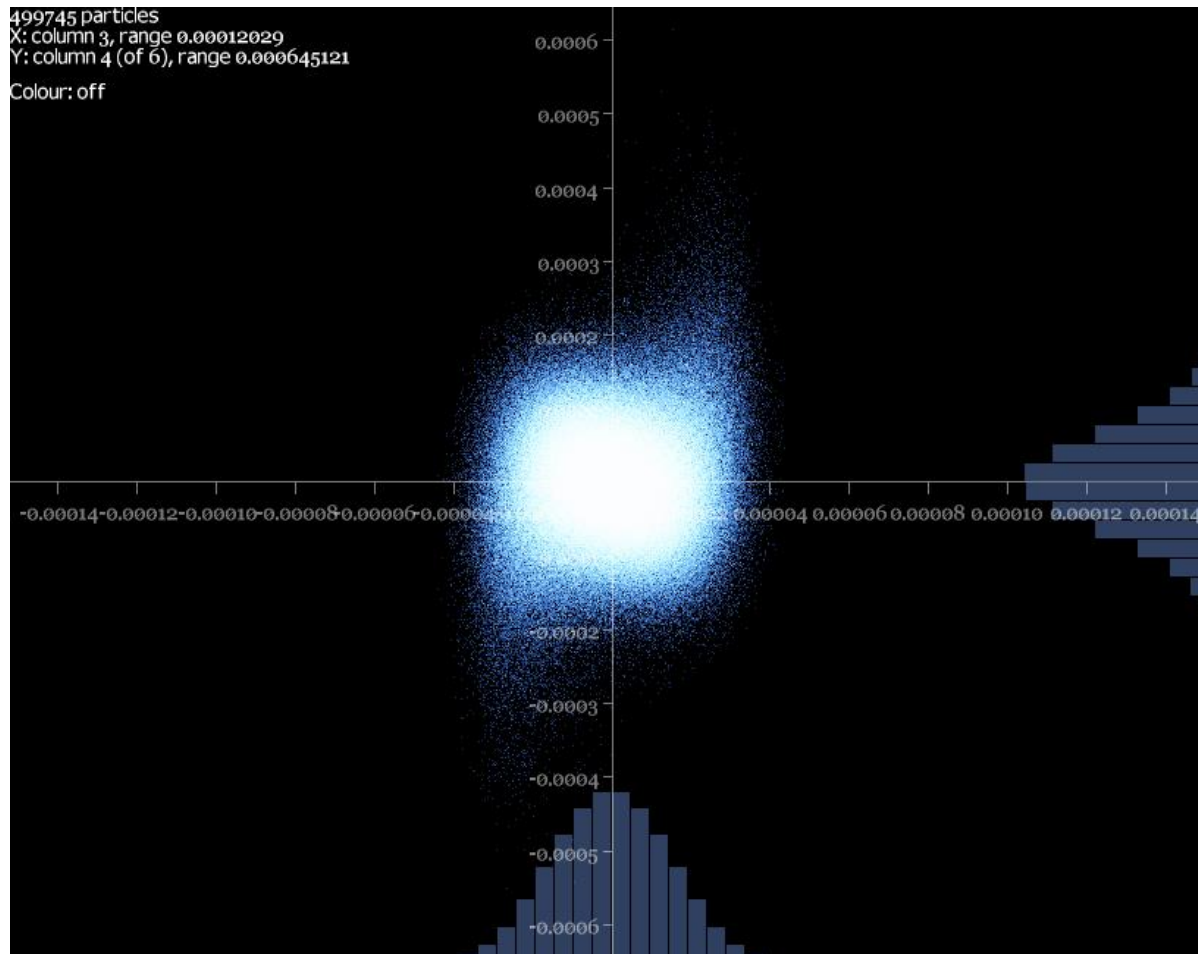
## Progress Report

# I. Beam Distribution Model

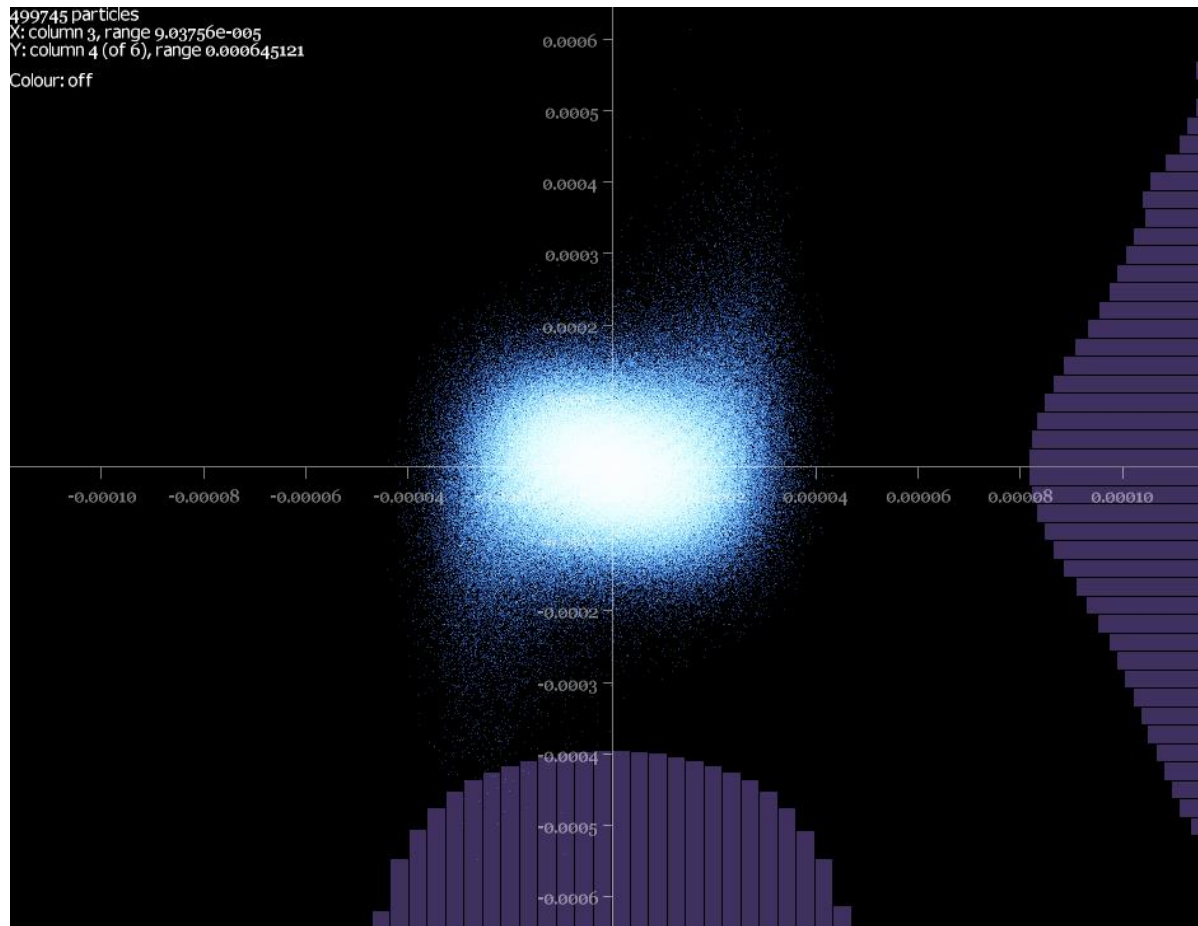
# Input from Beam-Beam Simulation

- To check dynamic aperture, want a “worst case” beam with tails
- Received a distribution from Yue Hao
  - Electrons immediately after interaction at 10GeV
- Loss required  $<1e-6$  so long tails are important
  - Define model distribution not discrete particles
    - Fit model to Yue’s beam distribution

# $y$ - $y'$ with Linear Histograms



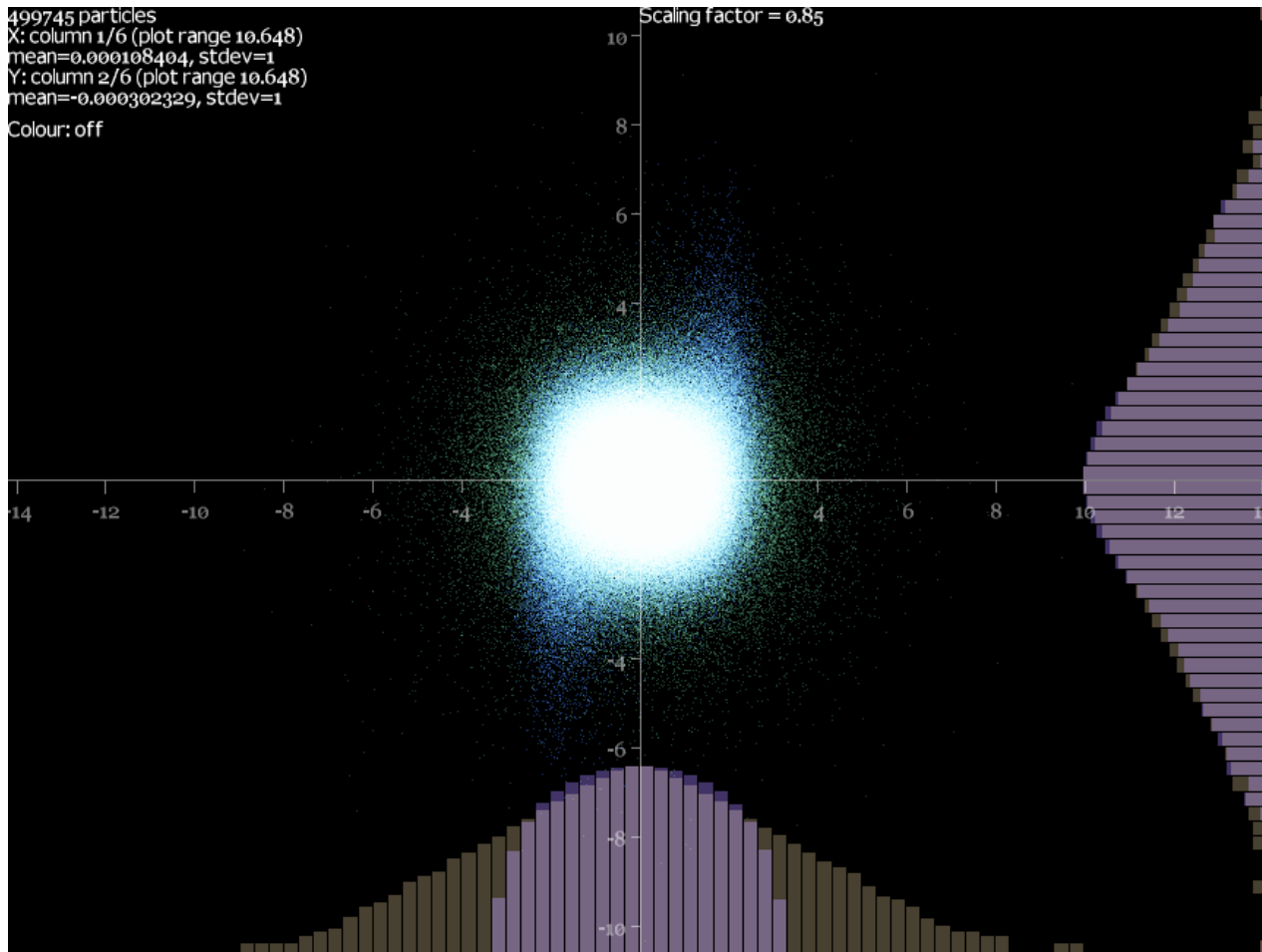
# $y-y'$ with Log Histograms



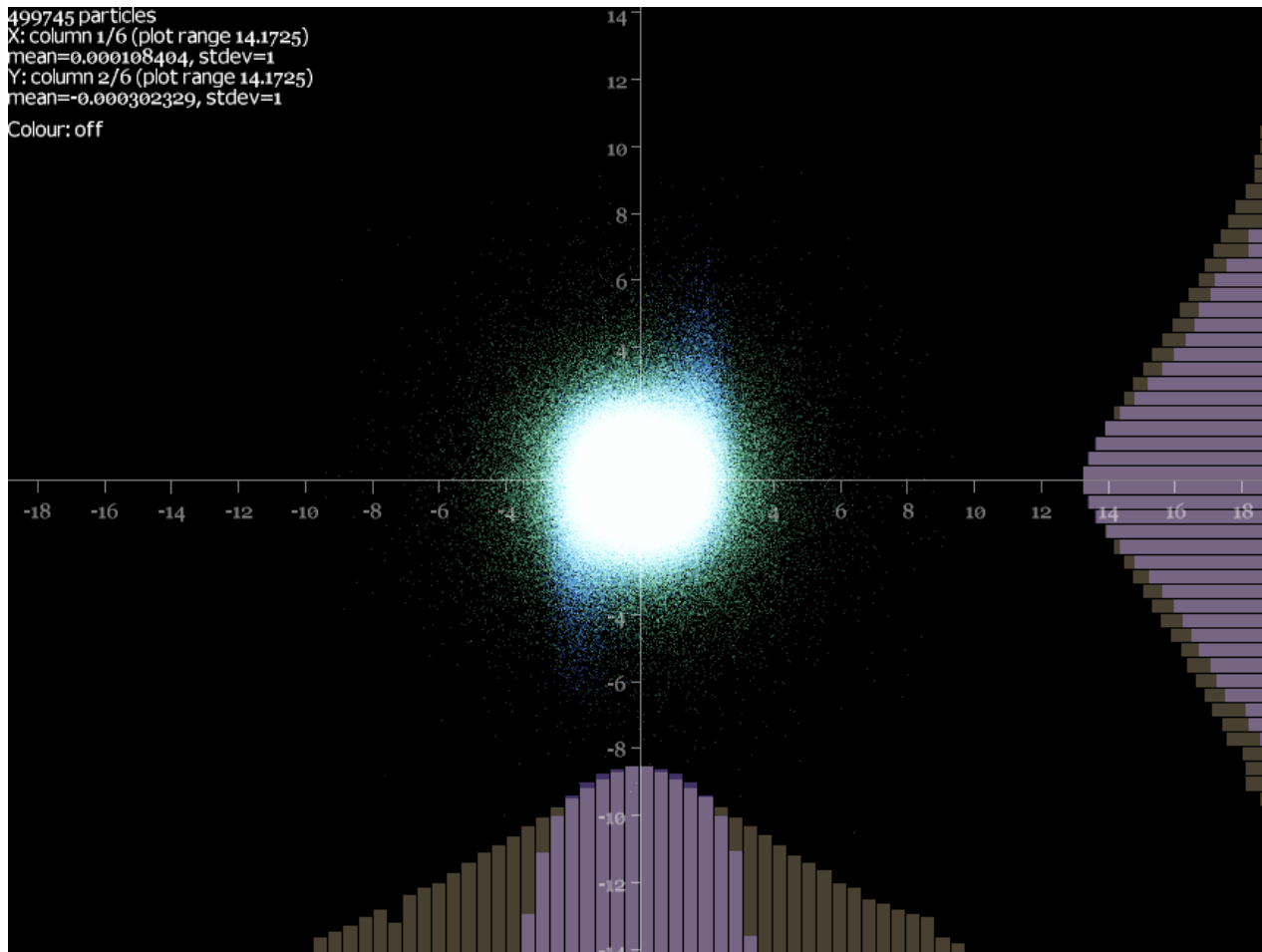
# Distribution Model

- $y'$  (and  $x'$ ) tails on log plots look like  $e^{-x}$
- Turns out in a 4D  $(x, x', y, y')$  distribution that's not a very natural tail to have
- Cumulative  $F(Z) = (1 - e^{-Z})^4$  mostly works
  - $Z$  is normalised amplitude (expressed in sigmas)
  - Needed to scale to  $0.85Z$  to get good fit in tails
- Idea is phase-independent distribution that is “at least as bad” as real one in all projections

# Distribution Model Comparison



# Comparison Without 0.85 Scaling

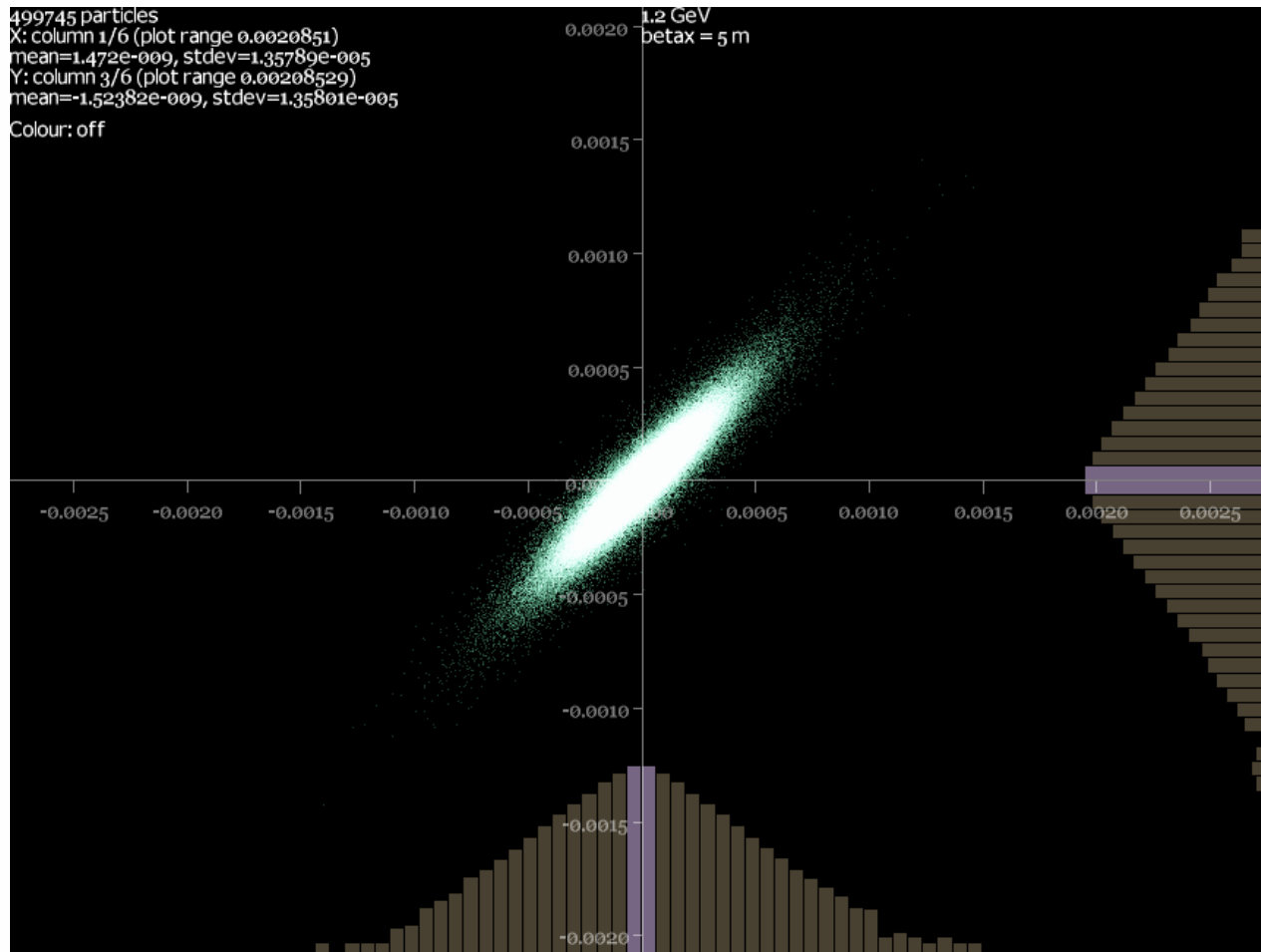




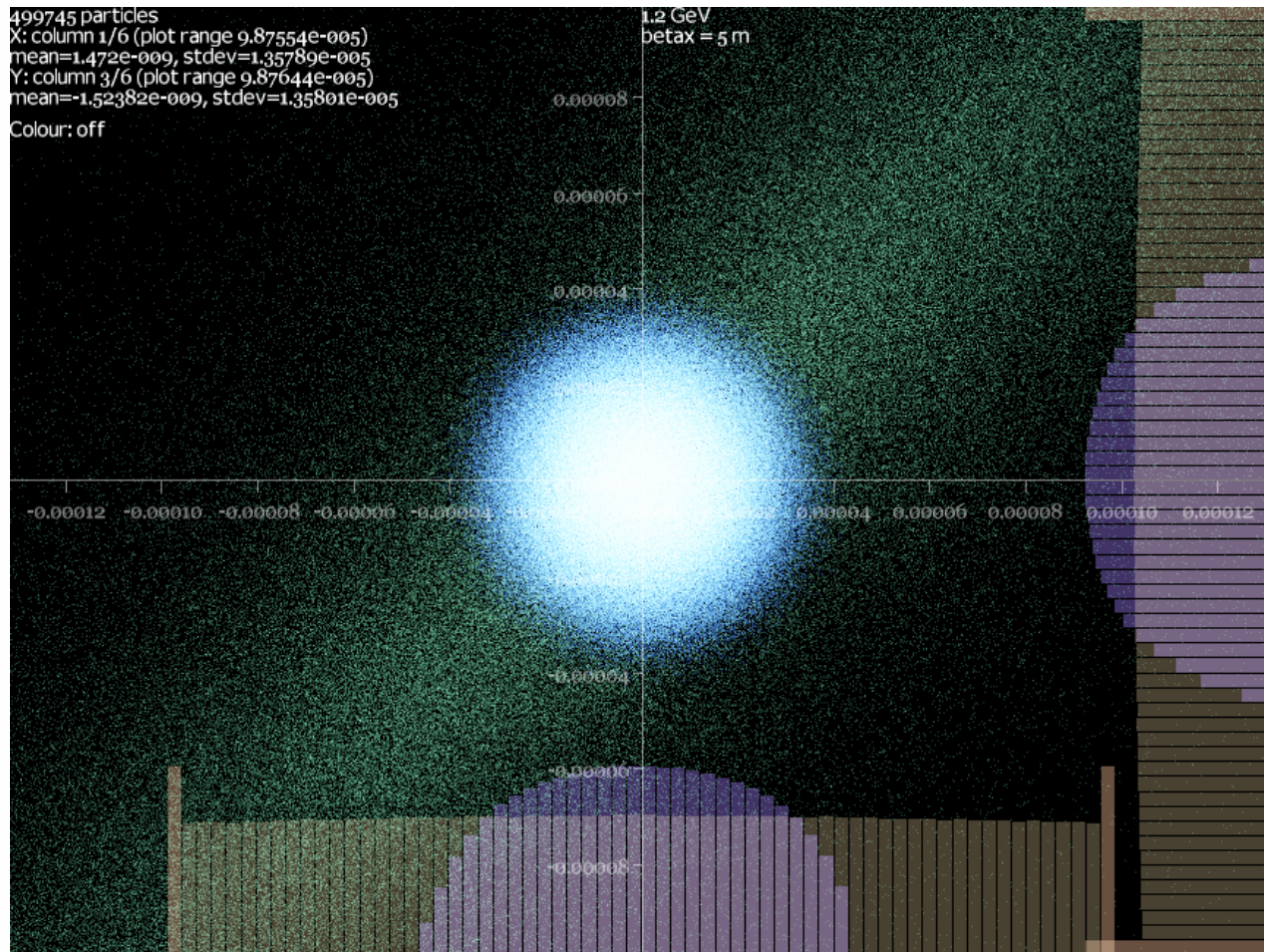
# Beam Size in Real Phase Space

- $\sigma_x = 13.6\mu\text{m}$ ,  $\sigma_{x'} = 72.9\text{mrad}$ 
  - At IR! So  $\beta_{\text{RMS}} = 18.6\text{cm}$
- $\text{em\_geom\_RMS} = 0.99\text{ nm.rad}$  (at 10GeV)
  - $\text{em\_norm\_RMS} = 19.4\text{ }\mu\text{m.rad}$
- Scaling to  $\beta = 5\text{m}$  and 1.2GeV gives
  - $\sigma_x = 203\mu\text{m}$
  - Tails were approaching 10 sigma
    - So [dynamic] aperture needs to be +/- 2mm: large!

# x-y Example VFFAG Beam @1.2GeV



# Zoom 20x to see Beam at IR

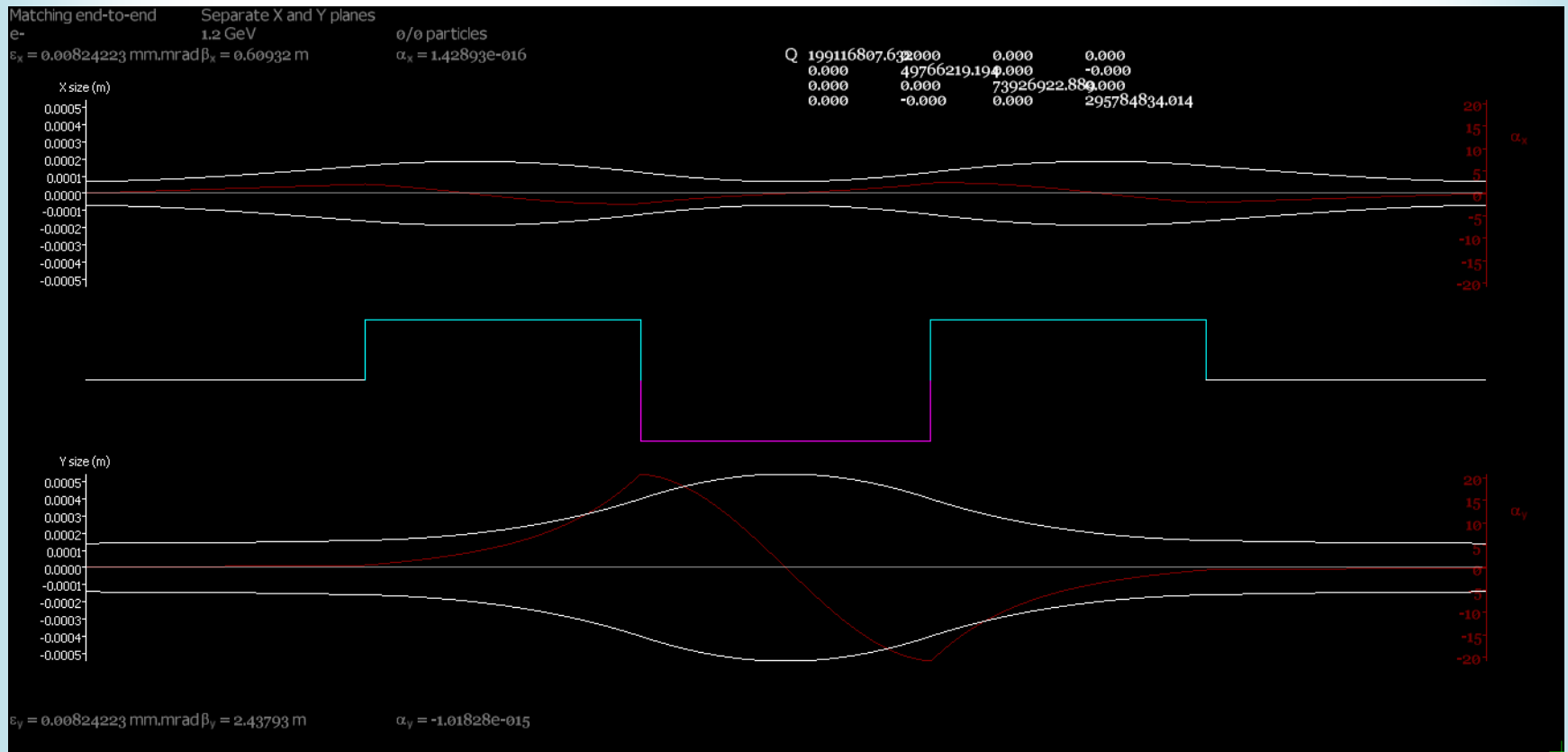


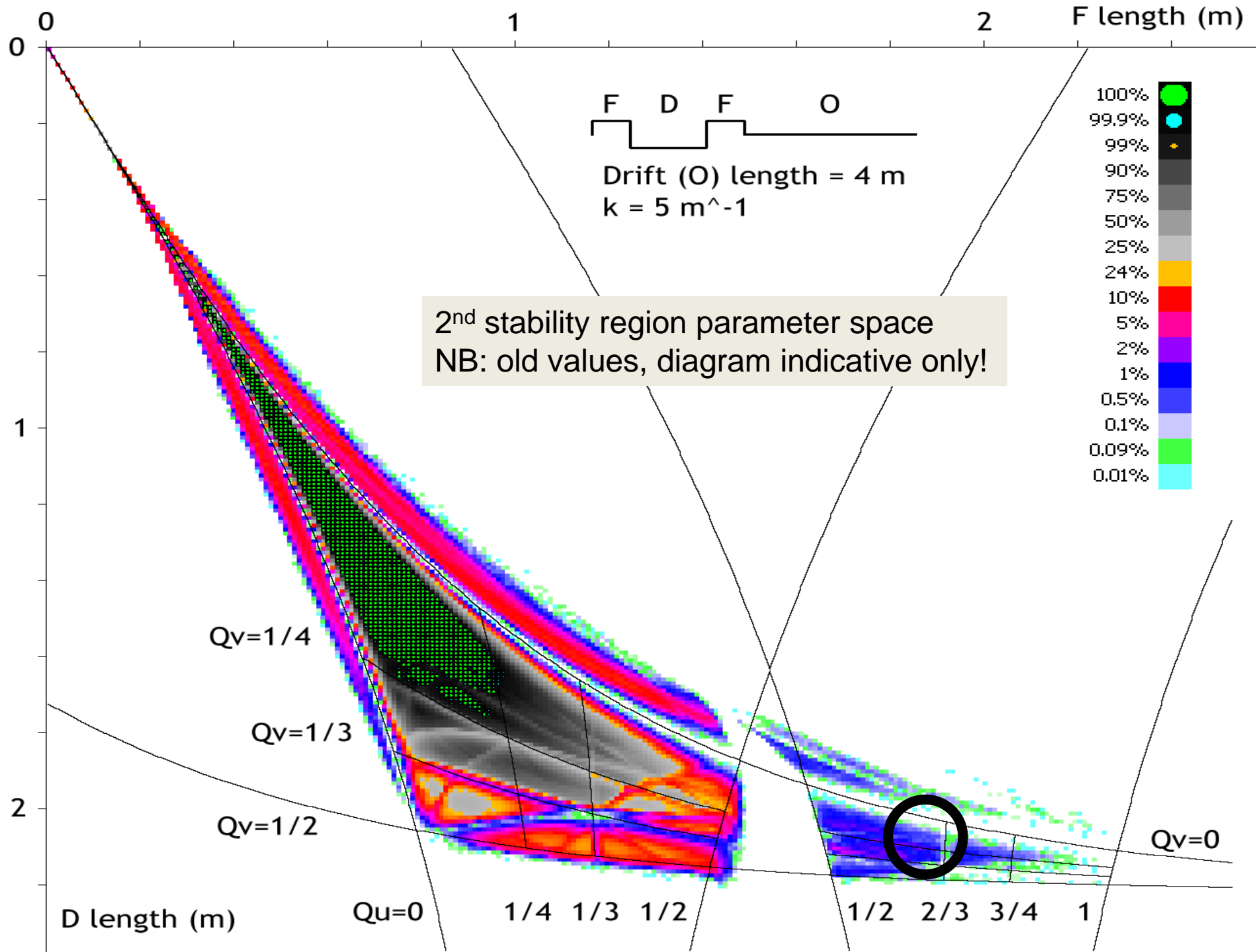
## II. Dynamic Aperture k Scaling

# Scaling VFFAG Design from April

- $k=100\text{m}^{-1}$  (reminder:  $B_y = B_0 e^{ky}$ )
  - 21.2mm orbit offset,  $B_0=0.0529\text{T}$ ,  $B_{10\text{GeV}}=0.441\text{T}$
- Used triplet “2<sup>nd</sup> stability region” lattice
- Tracking indicated dynamic aperture too small
- $F=1.23\text{m}$ ,  $D=1.3\text{m}$  (2cm fringe),  $O=2.507\text{m}$ 
  - FDFO lattice cell = 6.267m
  - 60% packing factor
  - Circumference factor = 3.241

# 2<sup>nd</sup> Stability Region Optics





# How to Scale k?

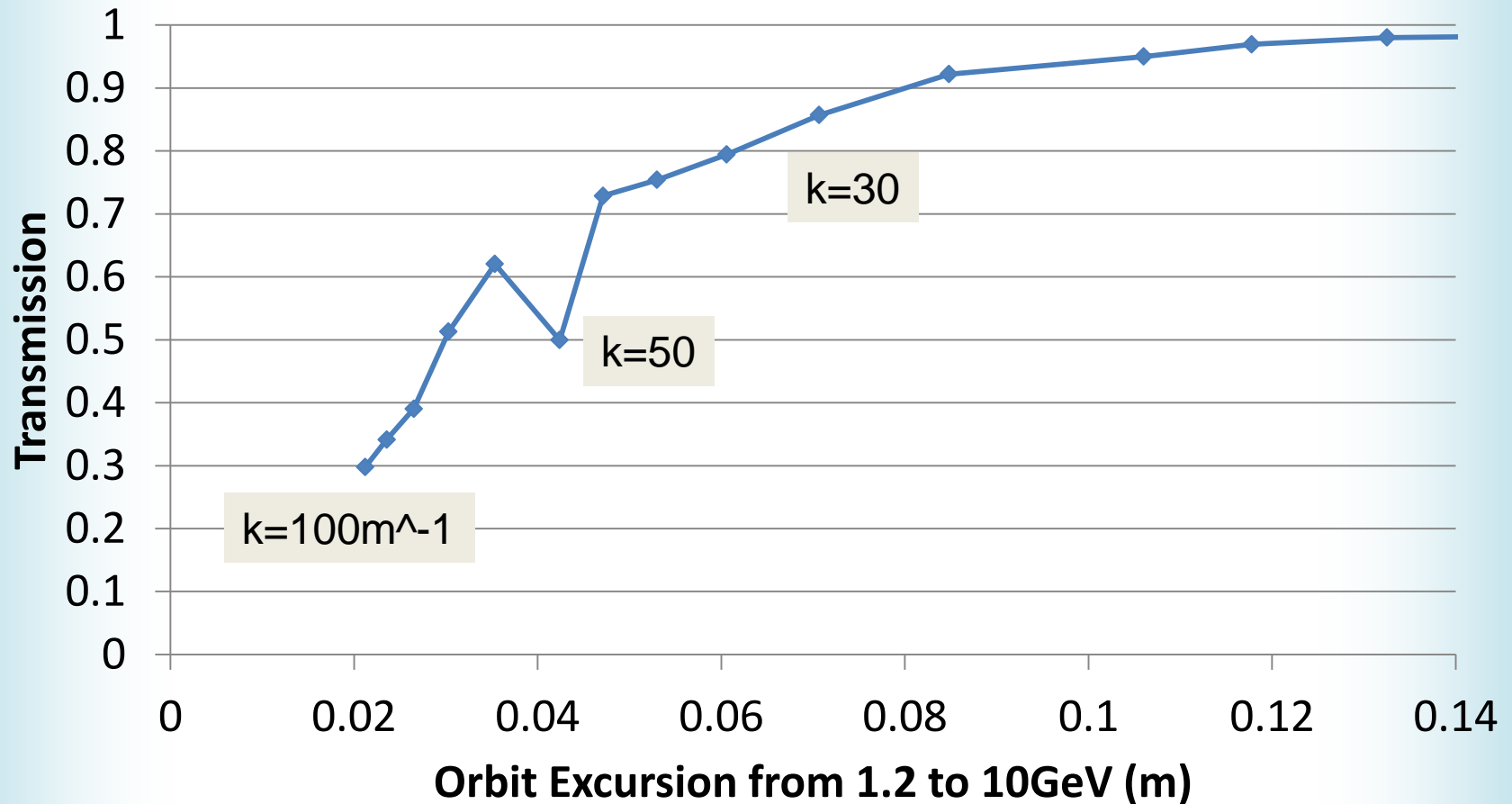
- Orbit spacing scales with  $1/k$
- Magnet/drift lengths with  $1/\sqrt{k}$ 
  - $2k$  with same magnet  $L$  gives focal  $L/2$
  - But  $2k$  with magnet  $L/\sqrt{2}$  gives focal  $L/\sqrt{2}$
- Beta is focal length so also  $1/\sqrt{k}$
- Fringe field length with  $1/k$ 
  - Because magnet height scales with  $1/k$
- Dipole fields remain the same



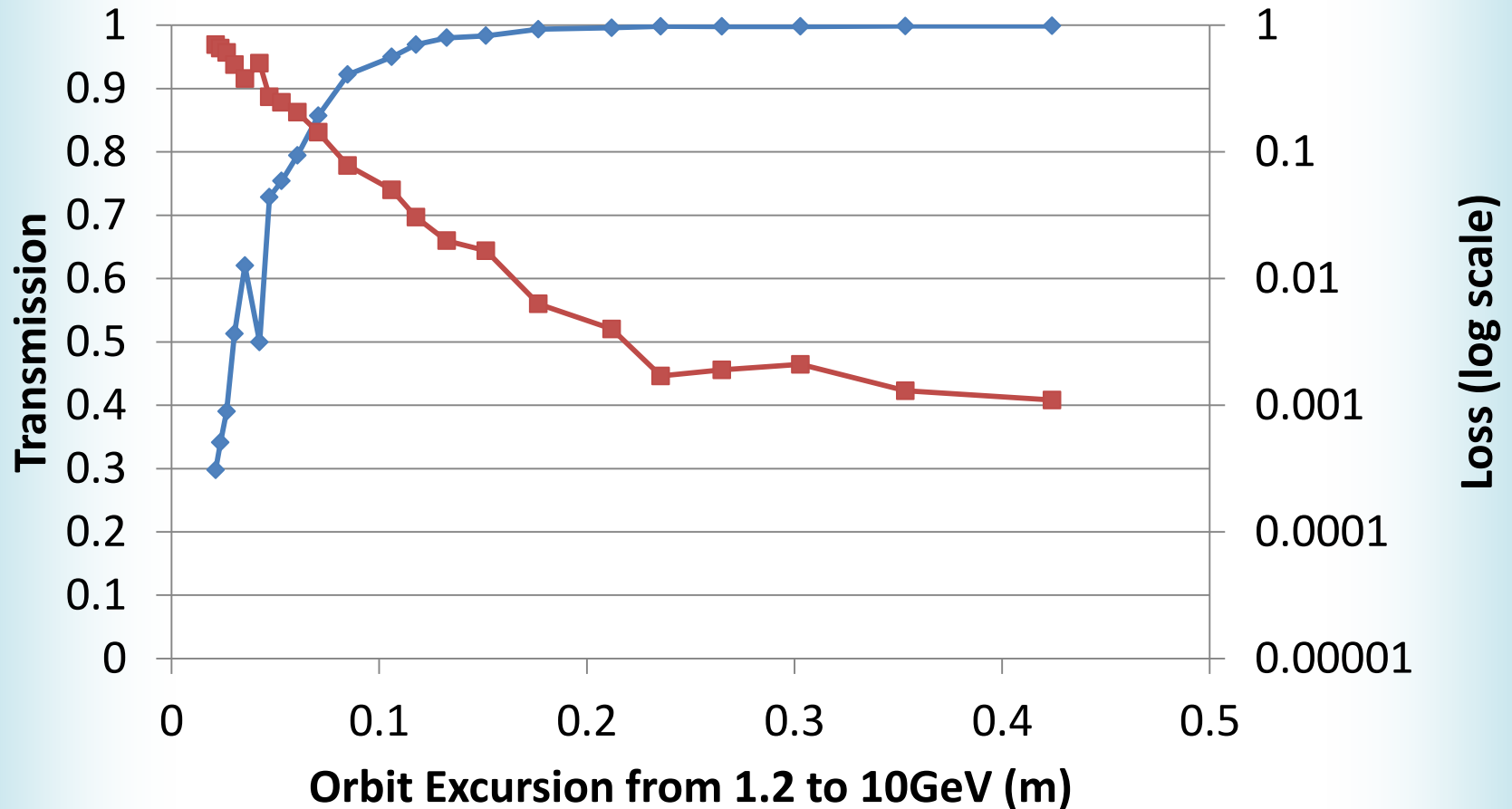
# Dynamic Aperture Simulation

- Most difficult turn is 1.2GeV: largest beam
  - Higher order poles smaller for smaller beams
- Disrupted beam is harder than original
- Do 2 turns of disrupted beam at 1.2GeV as proxy for whole 1.2->10->1.2 cycle
  - $C_{\text{eRHIC}} = 3843.16\text{m}$ , 2 turns  $\sim 7.7\text{km}$
  - 10000 particles for now
- 1.2-10GeV orbit excursion =  $2.12/k$

# Transmission as k is Decreased

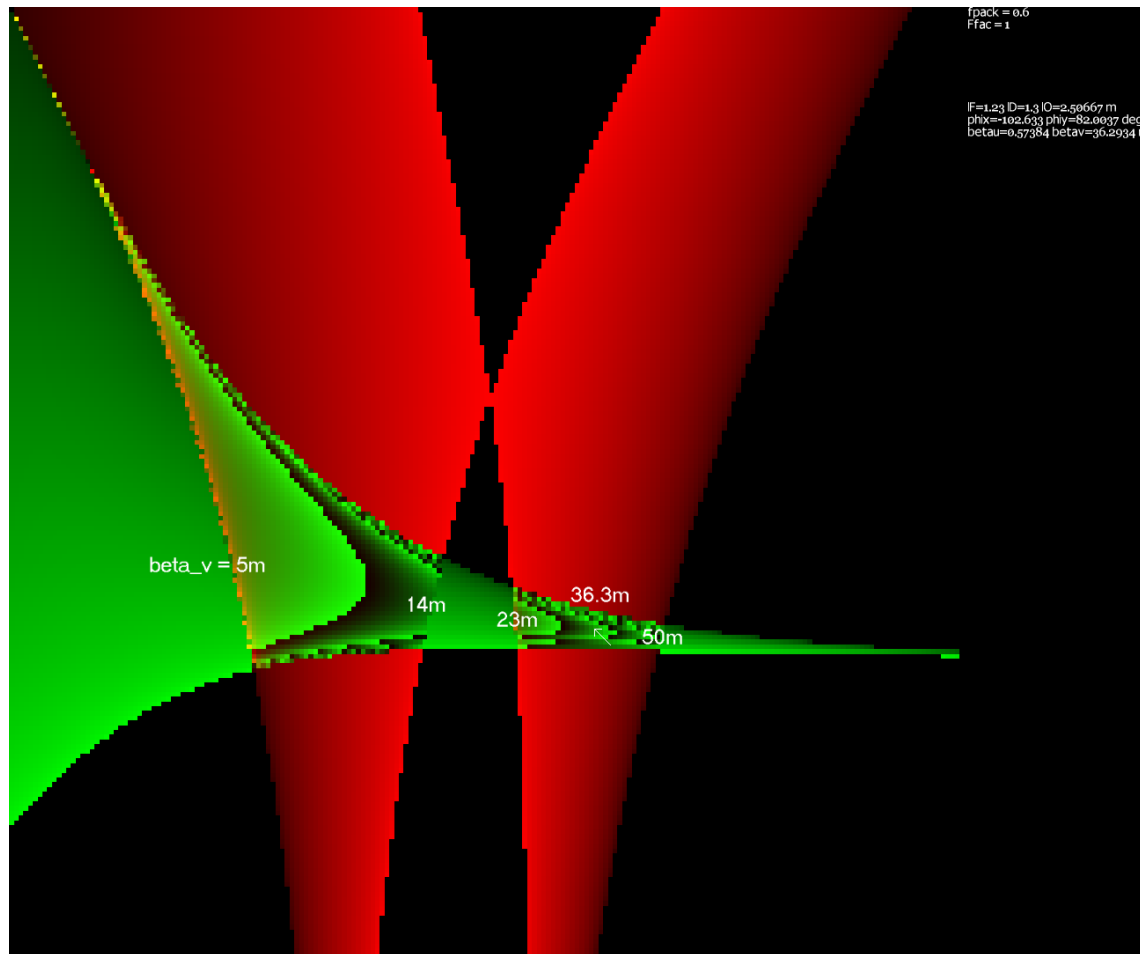


# Better but Not Good Enough!



# III. Stability Diagrams

# Necktie FDF Triplet Stability Plot



# Lattice Variables & Constraints

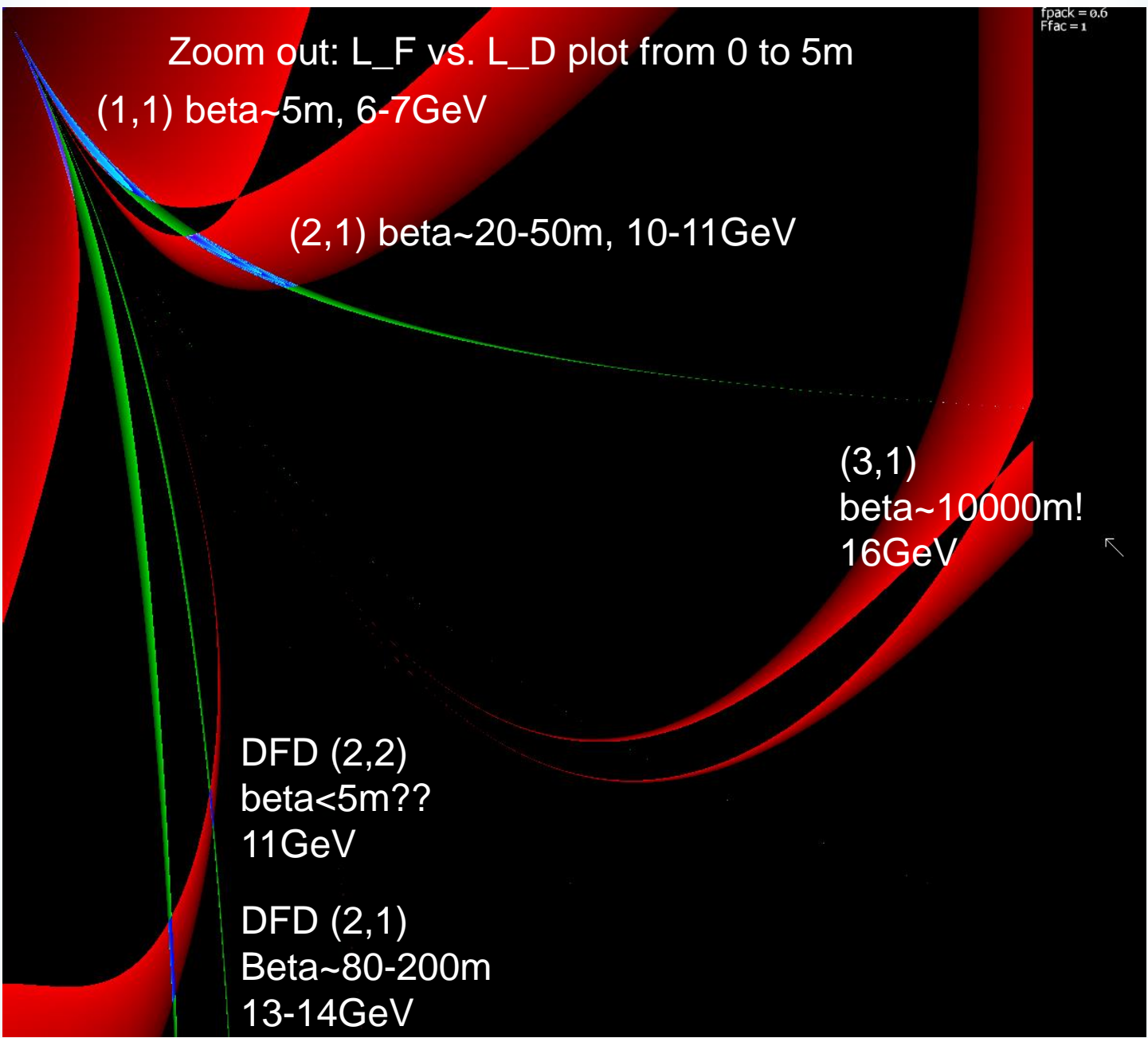
- FDFO triplet has 3 lengths and 2 magnet strengths (the  $B_0$  for F and D;  $k$  is fixed)
- Packing factor 60% fixes  $Length_0$
- Tunnel bending radius 378.26m fixes overall field magnitude
  - The previous “necktie” plots didn’t enforce this!
- Leaves  $5-2=3$  free parameters:  $L_F$ ,  $L_D$  and F/D strength ratio

L\_F vs. L\_D plot from 0 to 2m  
Curvature constant, 60% packing, strength F/D=1

FDF 1<sup>st</sup> stability region  
Max beta ~ 5m, 6-7GeV

DFD 1<sup>st</sup> stability region  
Similar to  
FDF 1<sup>st</sup>

FDF 2<sup>nd</sup> stability region  
Max beta ~ 20-50m, 10-11GeV





# IV. eRHIC Energy Limit Formula

# “Maximum GeV” for a Lattice

- Reference case: 60% packing factor,  $0.43^{2/3} = 0.2867T$ ,  $E_{ref}=20\text{GeV}$
- Synchrotron power  $P$  proportional to  $E^2B^2$
- $\langle P \rangle = E^2 \langle B^2 \rangle$
- So  $E_{max}^2 \langle ((E_{max}/E)B)^2 \rangle = E_{ref}^2 \langle B_{ref}^2 \rangle$
- $(E_{max}^4/E^2) \langle B^2 \rangle = E_{ref}^2 \langle B_{ref}^2 \rangle$
- $E_{max} = \sqrt{E E_{ref} B_{rmsref}/B_{rms}}$ 
  - $= E_{ref} \sqrt{E/B_{rms}} / \sqrt{E_{ref}/B_{rmsref}}$

# Maximum GeV Examples

- The non-FFAG eRHIC with separate beamlines (no reverse bend) has  $E_{\max}=20\text{GeV}$  and  $C=1$
- Having a circumference factor multiplies up the required fields ( $B_{\text{rms}}$ ) by  $C$  for a given energy, so  $E/B_{\text{rms}}$  is divided by  $C$
- Square root means  $E_{\max}$  is divided by  $\sqrt{C}$
- Existing  $C=3.241$  case gets to  $11.1\text{GeV}$

# V. Future Work

# Where to go next

- Try to find scaling VFFAG at lower energy ( $\sim 5\text{GeV}$ ) but with enough dynamic aperture
  - Perhaps alignment error studies too
  - Acts as a back-stop
- A few places remain to optimise the scaling VFFAG but since the tunes do not matter a non-scaling machine has more freedom