THREE-LENS LATTICES FOR EXTENDING THE ENERGY RANGE OF NON-SCALING FFAGs

S.J. Brooks*, RAL, Chilton, OX11 0QX, UK

Abstract

In this paper it is found that a three-quadrupole focussing system can be morphed continuously through FFD, FDF and DFF variants and back again while maintaining stable optics and even keeping the two transverse tunes constant. This relates to non-scaling FFAGs, where the magnet gradients define both the focussing and the field variation as the closed orbit position changes with momentum. A two-lens focussing system cannot change the sign of either gradient without becoming unstable, meaning non-scaling FFAGs built with such a lattice eventually encounter too large a magnetic field at low energies. However, a theoretical system of magnet field variations using three lenses, with a potentially unlimited energy range and fixed tunes is presented here.

TWO-LENS FFAG MACHINES

Several variants of FFAG accelerators are possible, which by definition must share the properties of fixed magnetic fields and 'strong' alternating-gradient focussing. The earliest machines [1] used a $B_y \propto r^k$ scaling law to define the magnetic field and thus became known as *scaling FFAGs*. The advantage of that particular scaling law is that the beam optics only differ in scale at different beam energies, similar to a synchrotron, meaning both horizontal and vertical tunes are constant. Unfortunately to generate alternating gradients using this scaling law (without using spiral edge focussing), the field must change sign entirely for some magnets, giving reverse bending as well as defocussing, which increases the machine circumference.

Non-scaling FFAGs of various sorts were tried to alleviate this problem, usually resulting in varying tunes, but having all magnets bending in the correct direction, at least at the highest energy. The magnets may even be made completely linear as in the EMMA [2] machine. However, the majority of FFAGs still use two magnet types per cell, which means that one magnetic 'lens' must always be focussing and the other defocussing, to maintain stable optics. This is illustrated in figure 1 for a variety of machines. This puts a limit on the energy range of two-lens non-scaling FFAGs with no reverse bends at some energy, since one magnet type must continue to increase its field magnitude even as the beam momentum tends to zero.

Vertical Orbit Excursion and Variants

Another FFAG variation (the *VFFAG*) may be obtained by placing the increasing B_y fields in the vertical, rather than the horizontal direction. This still produces stable optics but with skew quadrupole focussing and the closed orbit moving upwards with energy. A *scaling VFFAG* exists [4] (originally called a "ring cyclotron" [5]) with analogous properties to the scaling horizontal machine using the field law $B_y \propto \mathrm{e}^{ky}$. The VFFAG has certain advantages if one wishes to use superconducting magnets with slotted apertures, regardless of whether they are scaling. In fact they do not have to be entirely vertical, as the closed orbit can be made to move along an x–y curve during acceleration, a property that may be used to enforce isochronicity.

To classify: FFAGs may have horizontal or vertical (or skew) orbit excursion and be either scaling or non-scaling, optionally with linear fields. Useful characteristics exhibited by some but not all FFAGs include: fixed tunes; unlimited energy range; isochronicity (or quasi-isochronicity where $\mathrm{d}t_{\mathrm{rev}}/\mathrm{d}p = 0$ only at one momentum) and lack of reverse bends at the top energy, for a smaller circumference. These properties are summarised in table 1.

Table 1: Classification of FFAGs and their characteristics. Uppercase 'Y' indicates property is always true, lowercase 'y' that it is achievable in some cases. '3+' means three or more magnet types are required. '?' means unknown.

Type of FFAG	**Fixed tunes**	**Wide E range**	**Isoch-ronous**	**Small ring**
Scaling	Y	Y	N	N
Non-scaling	3+	?	y	y†
Linear n.s.	N	N	y(quasi)	y
Vertical s.	Y	Y	N	N
V. n.s.	3+	?	?	?
Linear v.n.s.‡	?	?	?	?
Skew	y	?	y†	?

†Two 'y's may not be achievable simultaneously.

‡Linear field VFFAG suggested by D.J. Kelliher.

THE THREE-DIMENSIONAL "NECKTIE" DIAGRAM

Adding a third lens to FFAG cells has some promising features: there are now three focussing strengths but only two transverse tunes, so many lattices should be possible for each tune point. The magnetic gradient of the third lens could also potentially change in sign, provided the other two lenses had opposite strengths (defining a stable FODO lattice) at that energy.

The space of stable two-lens designs for a given set of drift lengths is neatly portrayed by the *necktie diagram* [6]

* stephen.brooks@stfc.ac.uk

of standard accelerator physics. The equivalent diagram for a three-lens cell will be a 'stable volume' in three dimensions and such a thing is plotted in figure 2. Unlike the situation with two lenses where a given $Q_{x,y}$ narrows the choice down to two lattices: one FD and one DF, the three-lens lattices at a given tune point have the topology of a one-dimensional ring. This opens the possibility that the focussing system in an FFAG may move around this ring as the beam energy changes, allowing a greater variety of magnet configurations while keeping the tunes fixed. This is especially useful since the sign of the focussing strength determines whether the magnetic field is increasing or decreasing, so non-monotonic field profiles are possible with three or more magnet types per cell.

OSCILLATING EXPONENTIAL MAGNETS

Combining the exponential field of the VFFAG with an oscillation to move the focussing system around the stable volume of figure 2 suggests field profiles

$$B_{y,n}(0,y) = B_0 \mathrm{e}^{ky}(1 + a\cos(wy + \varphi_n)),$$

where a and w control the amplitude and frequency of the oscillations and φ_n is a lattice pseudo-phase that varies between magnets, so they do not all focus simultaneously. This formula resembles a spiral focussing machine [7] but uses discrete rectangular magnets and has the horizontal r^k scaling replaced by its vertical analogue e^{ky}.

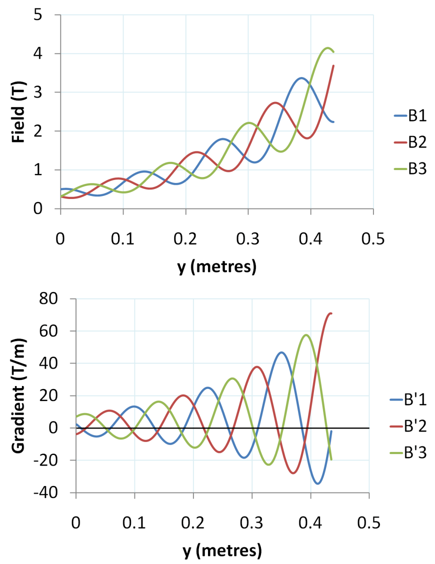

Figure 3: Set of three oscillating exponential magnets with $k = 5\,\mathrm{m}^{-1}$, $w = 50\,\mathrm{m}^{-1}$, $B_0 = 0.375\,\mathrm{T}$ and $a = 1/3$. The pseudophase $\varphi = 2\pi(n-1)/3$ for the n^{th} magnet. At least one of the gradients is negative at any position.

This section considers three-lens cells with $\Delta\varphi = 2\pi/3$ between consecutive magnets, whose field profiles resemble those in figure 3. The stable volume is thin in the $(1,1,1)$ direction so k must in fact be an order of magnitude lower than shown in the figure, giving a very slow field increase. This is because the e^{ky} term adds positive focussing to all three magnets since B_0 is always positive (no reverse bends, unlike scaling FFAGs), moving the lattice towards FFF, which is unstable.

Closed Orbit Sheet Behaviour

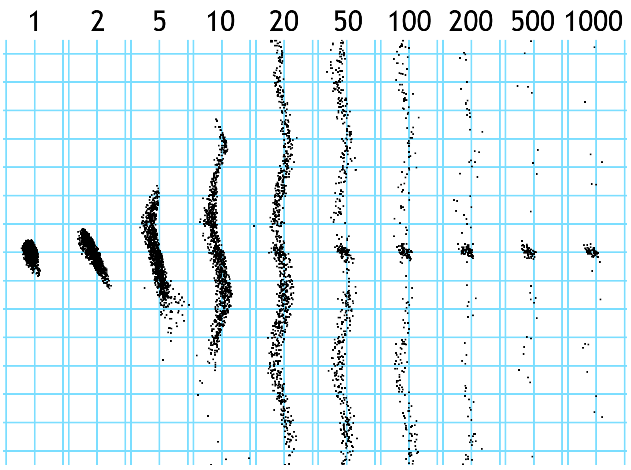

Figure 4: Weak vertical focussing behaviour in a lattice with three $k = 0$, $w = 12\,\mathrm{m}^{-1}$ oscillating exponential magnets. Grid is 10 cm in (x, y), numbers are cells tracked.

The low value of k led to a search for closed orbits in the $k = 0$ machine. This machine will not have a wide energy range but it does demonstrate a surprising result of changing the focussing system with height. Figure 4 shows that the beam experiences very little vertical focussing because there is a valid lattice (and closed orbit) for every y height, for the same energy. Several pseudo-phase cycles are visible as the beam diverges vertically, maintaining horizontal focussing and a small, oscillating closed orbit shift. Eventually a small core beam remains at the centre, which is vertically focussed by second-order effects.

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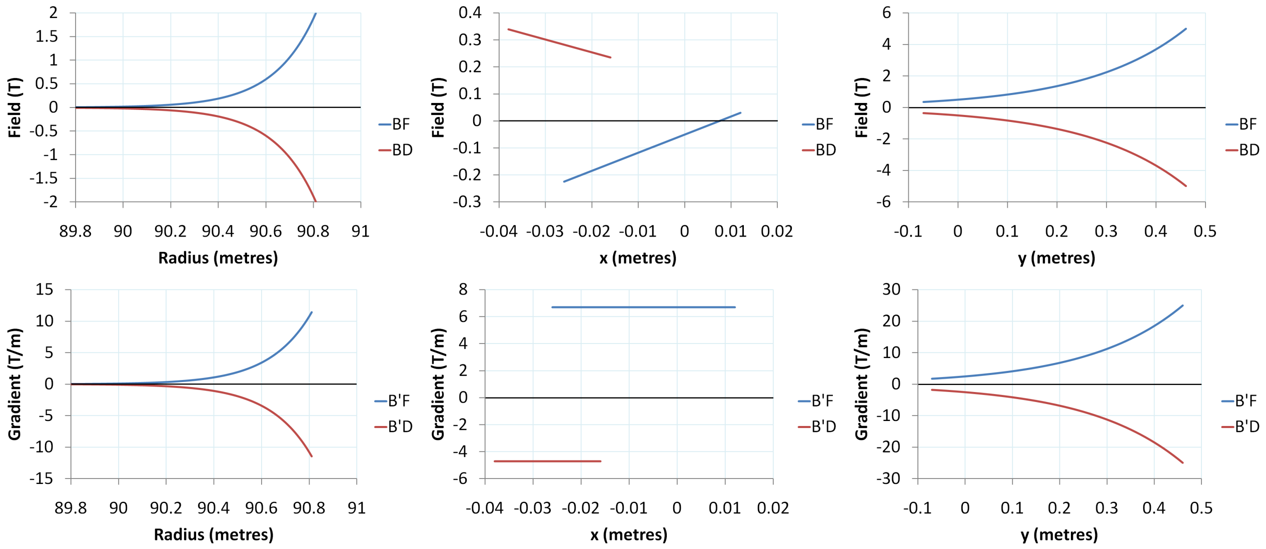

Figure 1: Magnetic field (B_y, top) and its derivative (bottom) in a scaling FFAG ([1], left), a linear non-scaling FFAG ([2, 3], centre) and a 'scaling' exponential field VFFAG ([4], right). Note B'_y never changes sign in these two-lens lattices.

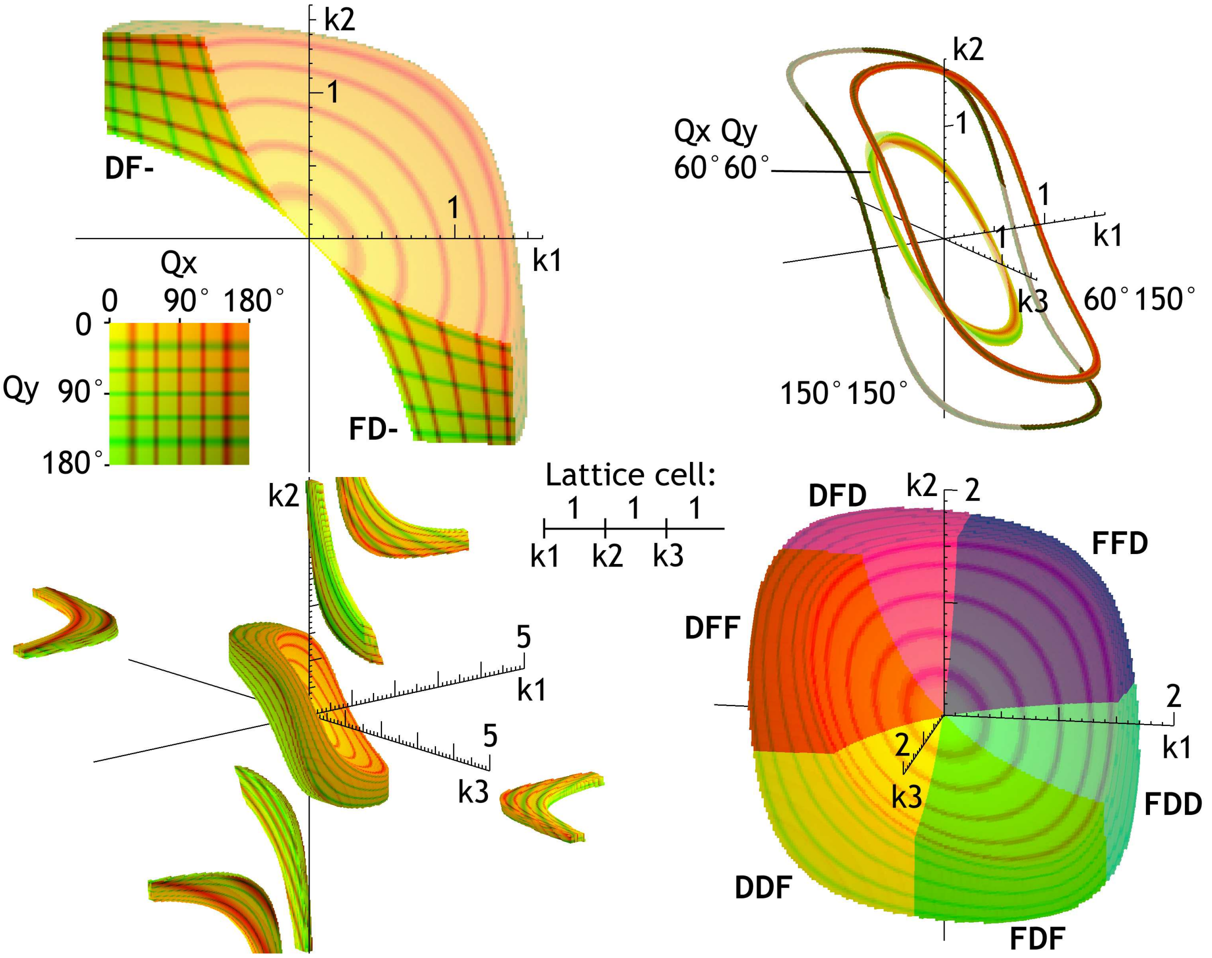

Figure 2: The three-dimensional stability diagram for a lattice with three quadrupole lenses of strength k_i separated by unit drifts. Top left: cut-away of the stable volume in the $k_3 = 0$ plane reveals two-dimensional necktie diagrams, coloured by cell tunes $Q_{x,y}$. Top right: loci of constant tunes within the volume. Bottom left: zooming out reveals secondary stable regions with tunes above π. Bottom right: colouring by signs of the three lenses reveals a cycle of six lattice types.