VERTICAL ORBIT EXCURSION FFAGs

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Abstract

Fixed-field strong focussing accelerators (FFAGs), in which the beam orbit moves with increasing momentum into higher field regions, have been widely studied. Less well-known is that the central orbit does not need to move outwards with energy: it can move in any direction including the vertically-moving orbit discussed in this paper. This allows for a magnet design with a smaller magnetised volume for a larger total energy range. A vertical analogue to the scaling FFAG is defined and its dynamic aperture studied for the case of an energy booster to the 800 MeV ISIS synchrotron [1] at RAL with various possible lattices.

MAGNETIC FIELD FOR VERTICAL ORBIT EXCURSION

Many magnetic fields permit vertical orbit excursion: if a dipole field (B_y) exists at $y = 0$, the central orbit will move to paths where $B_y\ell$ is larger as momentum increases in order to close the orbit. Thus if the magnetic field for $x = 0$ and $y > 0$ is a pure B_y component that increases with increasing y, then closed orbits for higher momenta will exist moving up the y axis. It is a misconception to think that 'centrifugal force' moves the orbit outwards as momentum increases: in fact the beam adiabatically tracks the closed orbit provided it has stable optics. In the vertical excursion case, any initial outward movement from centrifugal force moves particles horizontally into regions where the vertical gradient implies there is a B_x component, which then pushes the particles upwards as required.

This paper concentrates on what could be termed the *scaling* vertical-orbit-excursion field by analogy with (horizontal) scaling FFAGs. The magnetic fields in scaling machines are derived from a symmetry principle. If a closed orbit is scaled by a factor a in size (and possibly translated or rotated) while magnetic fields on that orbit are scaled by b, then the beam momentum $p \propto B\rho$ must scale by ab. Scaling FFAGs use a group of transformations that scale about the ring centre with $b = a^k$ [2] for some *field index* k. Vertically scaling FFAGs as defined here use a group of translations for which $a = 1$ but $b = e^{k\Delta y}$ for vertical orbit offset Δy.

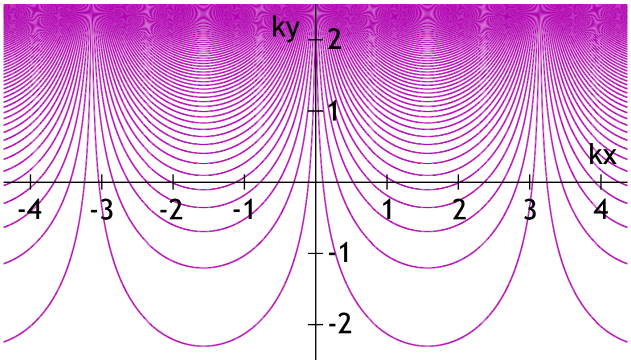

Figure 1: General form of a two-dimensional magnetic field that varies exponentially with y.

The field in a long magnet (with no field variation in z) that satisfies the vertical scaling condition can always be written, after a possible shift of x origin, as

$$B_y = B_0 e^{ky} \cos kx \qquad B_x = -B_0 e^{ky} \sin kx.$$

This field is plotted in figure 1: its strength increases exponentially with y and Maxwell's equations ensure that the field vector rotates with x. This stems from the one-to-one correspondence between 2D magnetic fields and complex functions, in this case e^{-ikz}.

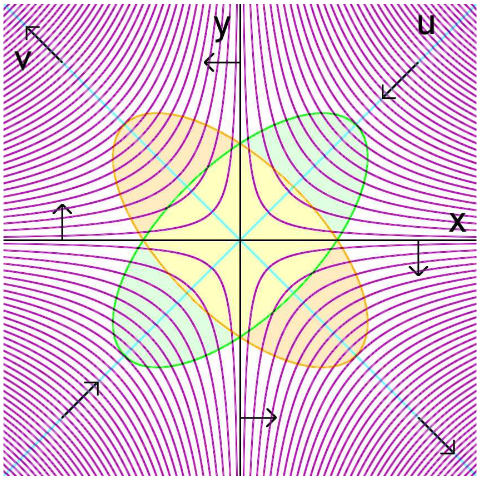

Figure 2: Skew focussing in the vertically-scaling FFAG field. The purple field lines cause the focussing forces (black arrows) along the u and v axes, giving beam cross sections that vary between the two elliptical shapes shown.

As in scaling FFAGs, this magnetic field is combined function, with a dipole and gradient (and all higher multipoles) superimposed. For small distances from the origin, the field is to first order

$$\begin{aligned} B_y &= B_0 + B_0 ky + \cdots \\ B_x &= - B_0 kx + \cdots, \end{aligned}$$

which produces skew focussing optics as shown in figure 2. Is it useful to define new transverse axes

$$u = (x + y)/\sqrt{2} \qquad v = (y - x)/\sqrt{2},$$

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which are rotated by 45°. Using these, the usual expression for a quadrupole field reappears:

$$\begin{aligned}
B_u &= (B_x + B_y)/\sqrt{2} = B_0/\sqrt{2} + B_0 k(y - x)/\sqrt{2} \\
&= B_0/\sqrt{2} + B_0 kv \\
B_v &= (B_y - B_x)/\sqrt{2} = B_0/\sqrt{2} + B_0 k(x + y)/\sqrt{2} \\
&= B_0/\sqrt{2} + B_0 ku.
\end{aligned}$$

Another important feature shared with scaling FFAGs is that optics are identical at each momentum, so tunes stay constant, which is important for proton machines with many turns or significant space charge. Also, the focussing is proportional to the bending, which has the unfortunate consequence of requiring extensive use of reverse bends in all alternating gradient lattices.

MAGNET DESIGN IMPLICATIONS

Producing a vertical magnetic field on a vertical orbit sweep can be easier than on a horizontal aperture, as required by conventional machines. Figure 4 shows that conductors placed symmetrically above and below a horizontal aperture slot, in order to produce a pure B_y field on orbit, actually cancel each other out at the nearest point on the plane. Normal conducting dipoles resemble split solenoids on their side for this reason, with the coil at the periphery of the beam region and field enhanced by iron.

Reversing the current in one of the conductors so that the fields are additive on the plane produces more field closer to the sources but now the field is horizontal. This problem can be turned into an advantage by rotating the whole situation so that both the field and aperture slot are vertical.

Figure 3 shows how such a magnet might be realised in practice, with 250 A/mm^2 current density for the lattice used later in this paper. The coils are generally thinner than those required in a horizontal orbit excursion machine because they are nearer to where the field is required. For small beams, the magnetised volume can be made very small by narrowing the gap between the conductors. As the conductors have opposite polarity, forces on the coils are outwards, which allows a large external support structure to be used if necessary.

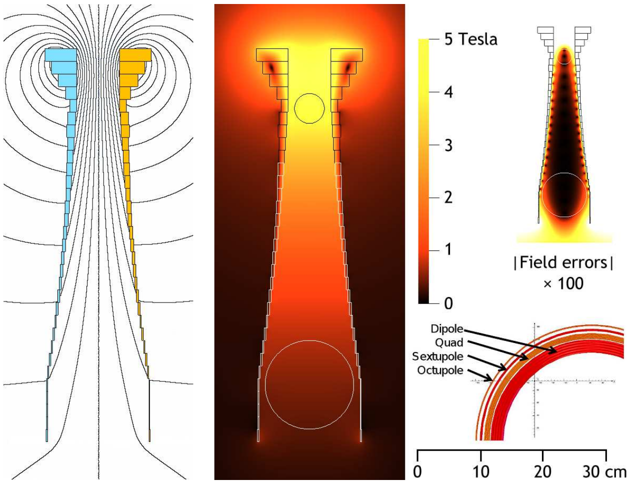

Figure 3: Cross-section of a superconducting magnet design that produces the vertical scaling field. Circles indicate outer bounds for the ISIS beam at 0.8 and 12 GeV. Bottom right is the PAMELA magnet for a horizontal orbit excursion, shown to scale, adapted from [3].

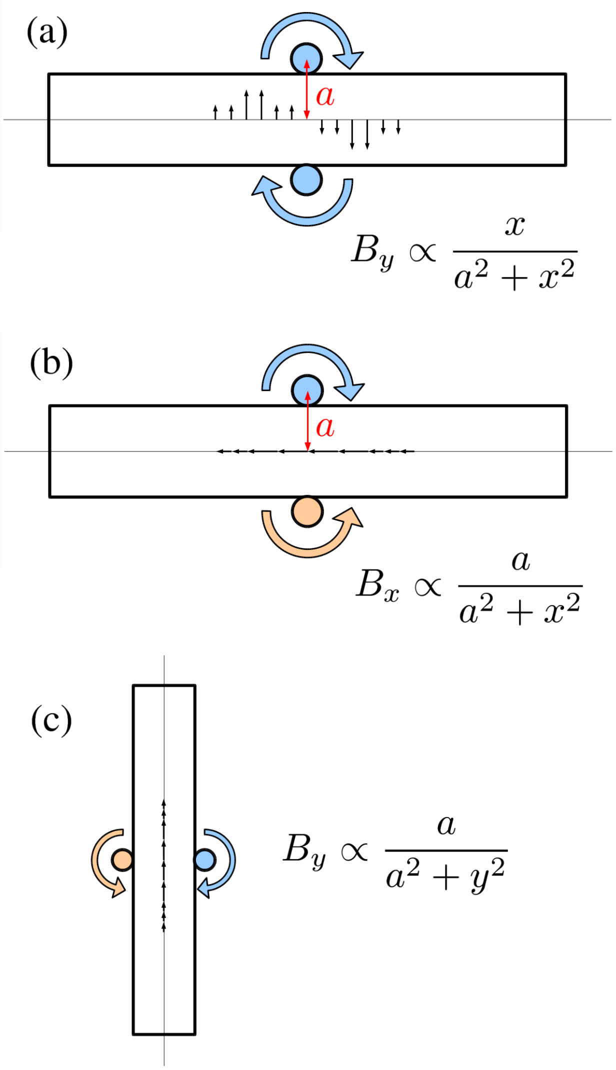

Figure 4: (a) Conventional method for producing a vertical dipole field on a horizontal slot aperture using conducting coils. (b) Reversing the direction of one current gives constructive interference but field in the B_x direction. (c) Rotating by 90° gives a vertical dipole field in a vertical aperture.

TRACKING

A vertical FFAG has been investigated for application as an energy booster for the ISIS synchrotron, starting with a bunch-to-bucket transfer of the existing 800 MeV beam and

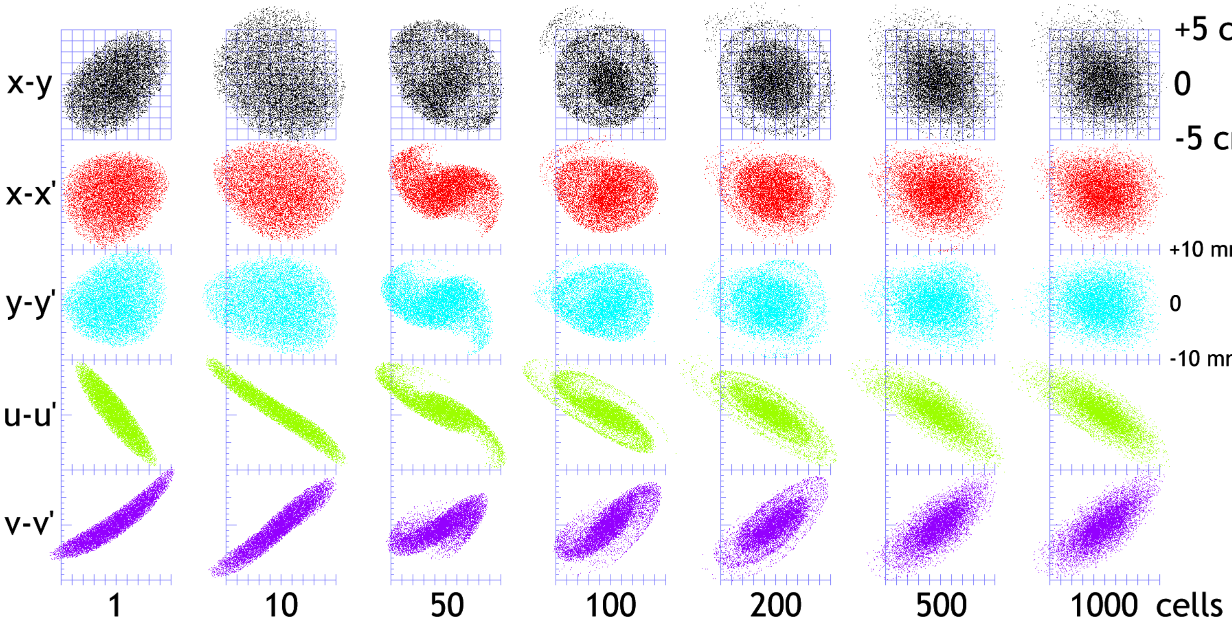

Figure 5: Tracking simulation of the FODO lattice showing real space, normal and skew phase space planes for increasing numbers of cells.

accelerating to 12 GeV with possible parameters shown in table 1. The use of superconducting magnets is desired to reduce the machine footprint but high mean power operation demands a 50 Hz acceleration cycle. As explained in [4], fixed field solutions allow both superconducting magnets and a larger time available for acceleration than a sinusoidally-varying field.

The final machine will have to be a non-scaling variant of the vertical FFAG, rather than the scaling one shown here, because the requirement to have roughly equal bends and reverse bends means the lattice would not form a closed ring for 10.9 km. The magnet packing factor of 9.6% would also have to be increased: it is low here because additional bending gives additional unwanted focussing unless k is reduced, which increases the orbit excursion.

A simulation of the transverse optics was performed with results shown in figure 5. The ISIS beam at extraction has a geometrical emittance of 150 mm.mrad and was injected as a circular waterbag beam with upright phase spaces into the middle of the drift in the FODO lattice (note that more sophisticated matching would require beam manipulation in the skew planes). The remaining parameter to choose was the Twiss beta at injection and it was found empirically that $\beta = 8$ m produced the best results. Different particles in the beam have different tunes due to the presence of higher-order multipoles, which causes spiralling in the skew phase spaces and it can be seen that the equilibrium phase space shapes are not entirely ellipsoidal.

An accelerating voltage was applied to the beam while it was simulated, producing the expected vertical motion shown in figure 6. The adiabatic shrinkage in beam size is also apparent, with size proportional to $1/\sqrt{\beta\gamma}$ i.e. $1/\sqrt{p}$, or in spatial terms $e^{-\frac{1}{2}ky}$.

Table 1: Parameters of the FODO lattice.

Energy range	800 MeV–12 GeV
Orbit excursion	43.5 cm (vertical)
k	5 m^{-1}
B_0	0.5 T
$B_{\max}$	4.41 T (beam centre) 4.96 T (beam top) 5.33 T (whole magnet)
Lattice	FODO
F length	0.4 m
D length	0.45 m
Drift length	4 m

DYNAMIC APERTURE

With nonlinear magnetic fields present in every element, it is sensible to ask whether the dynamic aperture of the machine can contain the beam. Simulations using 10000 particles and tracking through a length of 1 km (113 cells) were performed to test the beam transmission as the lengths of the F and D magnets in the FODO cell were varied, as shown in figure 7. The beam remained the same 150 mm.mrad one as before. Many resonances are visible in this diagram and they can only be avoided entirely by staying with fairly low cell tunes of below 0.2, with near-equal focussing in both planes also being a requirement.

One method that has been used by the PAMELA FFAG design study [5] to reduce the amount of reverse bending is to find the second stability region of a triplet cell where $Q > 0.5$. Figure 8 shows the beam transmission scan of an FDFO cell. The slanted line corresponding to equal fo-

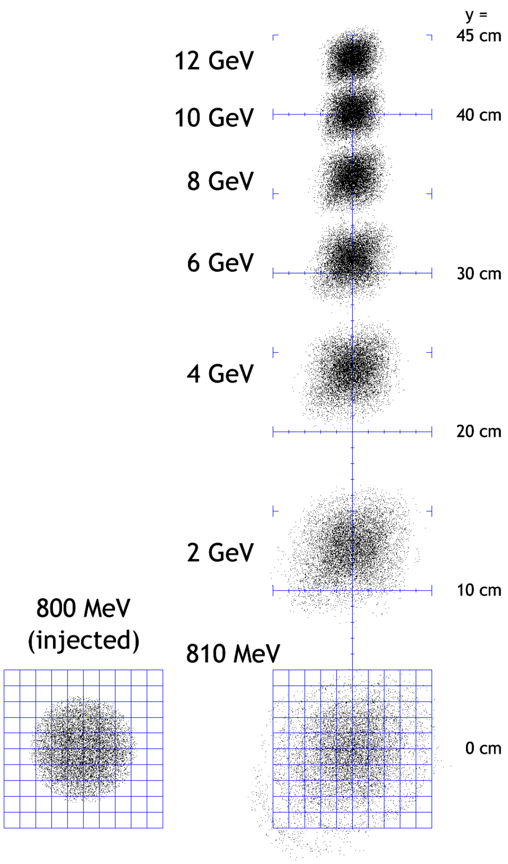

Figure 6: Acceleration of the 150 mm.mrad beam in real space, showing vertical motion and the increase in size of the beam as it matches to the nonlinear equilibrium phase space of the magnets over the first few MeV after injection.

cussing is prominent at the top left but the second stability region in the bottom right is quite far away from this line. Unfortunately the dynamic aperture of this region is not enough for the ISIS beam, as optics from the second stability region typically include large variations in beam size. The region is suitable for PAMELA because it is a proton therapy machine with a much smaller beam.

CONCLUSION AND FUTURE WORK

Without the availability of large superconducting magnets that can vary their field at high speed (e.g. 50 Hz), fixed-field machines will be pursued for applications that require high mean power or repetition rate. Applications that replace synchrotrons typically also require a fixed tune, which is guaranteed by a scaling FFAG (of horizontal or vertical type), though can be achieved with some care in a non-scaling machine. This paper combines the fixed field and fixed tune with a large dynamic aperture and a vertical orbit excursion that can reduce the magnetic volume required.

Currently the following features still need to be included for realistic simulation:-

- **Longitudinal dynamics**. This is expected to be mostly decoupled from the transverse motion, although it will modulate the space charge.
- **Space charge**. Although the peak space charge in ISIS is at 70 MeV injection and decreases rapidly with energy, a ring with larger circumference and strong resonances could make it a problem again at 800 MeV.
- **Field errors**. Any new sort of magnet and optics must be tested for its sensitivity to field errors.

Additionally, non-scaling variants of the vertical FFAG must be pursued to give a practical ring circumference, which will require fixed tunes to be enforced by some other method.

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[4] *Extending the Energy Range of 50 Hz Proton FFAGs*, S.J. Brooks, Proc. PAC 2009.

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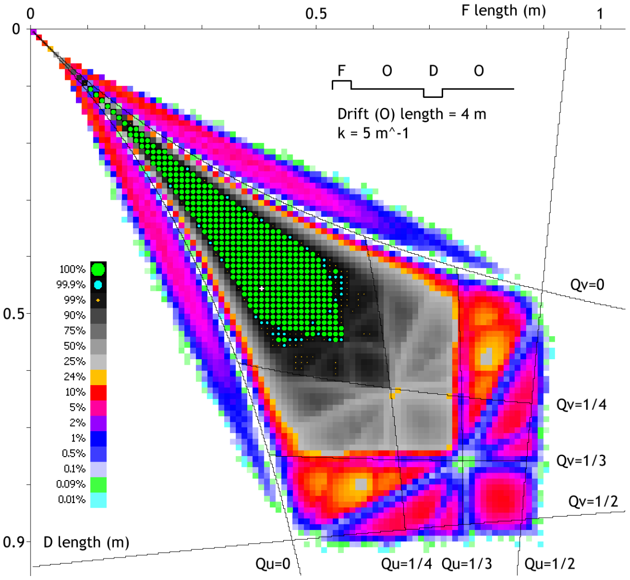

Figure 7: Effect on beam transmission of the lengths of the F and D magnets in the FODO cell. Magnetic fields are kept constant so this varies the total amount of focussing. Cell tune lines in the u and v skew planes are shown.

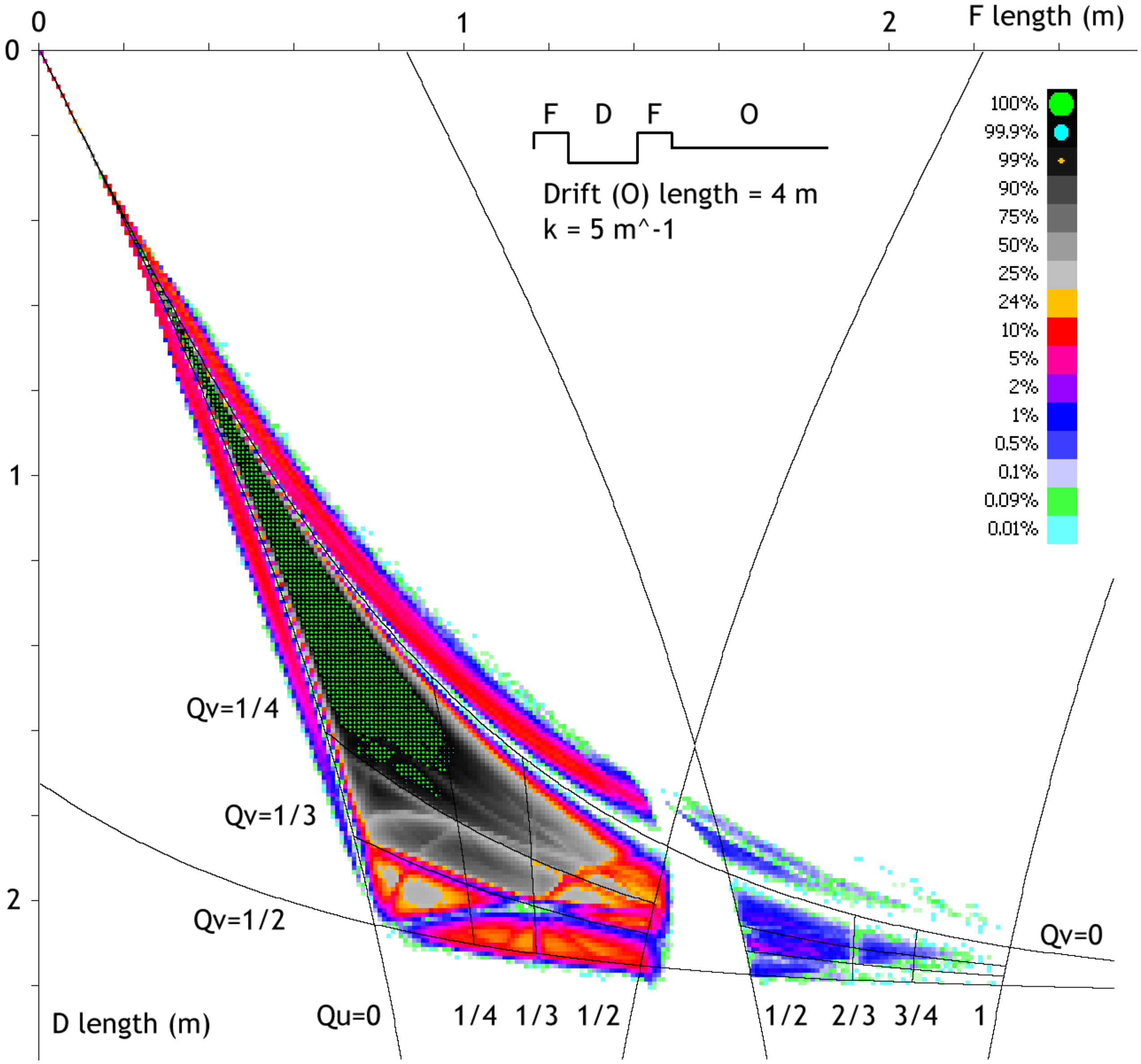

Figure 8: Beam transmission as a function of F and D element length in a triplet cell. The second optical stability region is visible to the right, though it does not have enough dynamic aperture for all of the beam. Bands of partial transmission outside the theoretical stable regions are residual unstable particles that have not yet been lost at 1 km distance.