Editor Corrections

PROBLEMS WITH MANUSCRIPT:

The following problems were noted in your manuscript upon receipt. These should not hinder the review process. Please do not send a revised manuscript or figures at this time.

* We have modified your title so that it conforms more closely to the style of this journal. Please check our suggested changes. Further changes can be made at any time during the review or production process.

Fixed in .tex file

* The use of "et al." in references is discouraged since the full list of authors can be helpful to our readership.

Expanded the full list of authors in all cases except for [19] where there were about 50 authors.

Figure Corrections

PROBLEMS WITH MANUSCRIPT:

In reviewing the figures of your paper, we note that the following changes would be needed in order for your figures to conform to the style of the Physical Review. Please check all figures for the following problems and make appropriate changes in the text of the paper itself wherever needed for consistency.

Figure(s) [7,9,12]
Do not spell out the names of Greek characters; please use the Greek letter.

Figure(s) [7,12]
Please modify quantities so that superscripts and subscripts are set as they would appear in text. We do not print computer notation. For example, set powers as superscript numbers; do not use ^2, **2 etc. for squares of quantities. Please adjust the text of the paper accordingly.

Figure(s) [7,9,12-18]
Please remove background grid and/or shading from your figures.

Figure(s) [1,11]
The size and font of the letters and numbers appearing in the

figure should be uniform (or close to it). Please make the letters and numbers more consistent.

Figure(s) [9]
Please fix the spelling error(s) in your axis label text.
Here preferred spelling: meters.
All of the above problems fixed in the figures

Referee A Corrections

This is an interesting paper exploring a novel accelerator concept, in which particles are accelerated using a horizontal ring of magnets with fixed but vertically increasing field strength, so that they travel in a helical path at essentially constant radius. This makes possible isochronous (and therefore cw, high-intensity) operation, as in cyclotrons.
The principles of such a scheme were explored by a few authors between 1955 and 1961, but not pursued. The present study has been carried out in much greater depth, and conceptual designs are presented for 3-, 5- and 12-GeV proton drivers to accelerate 200-µA beams from the ISIS synchrotron, backed up by simulations of transverse and longitudinal beam dynamics, spacecharge effects and superconducting magnet field distributions. A final section briefly reviews a number of other possible applications.

Minor comments

I, para.2: As "centrifugal force" is itself a misconception, it would be better to replace it in both its occurrences by "reduced orbit curvature" or something similar.
Rephrased para.2 to talk about "moving to larger radii" and "increased beam rigidity".
Also, the coordinate system should be specified – neither the x nor the z-axis is currently defined.
Now done at the end of para.1.
Here and in II.B, para.1: "relativistic" should be replaced by "adiabatic" – the shrinkage is caused by increased longitudinal momentum, not by Relativity.
Changed to "adiabatic". Was getting confused with longitudinal phase space where relativistic length contraction does occur.
I.A, para.1: The symbol ℓ(script l) should be defined or explained.
Added closed orbit "of length ℓ".
I.A, para.2: The use of the term "x= 0 mid-plane" here is a little disconcerting, given that the orbit is horizontally curved. Maybe there should be an explicit statement that straight rather than curved magnets are being considered.
Added "This paper assumes the orbit curvature within each magnet is small so field expansions treating $x=0$ as a flat `mid-plane' are valid."
II.A, Fig. 3(a): The arrows should vary smoothly in height.
Fixed.
II.B, Fig.5 (Centre): Even if the figure shows a nominally hard-edged beam, the clearance between it and the superconducting coils appears inadequate, especially for 200-µA (1.2 MW).
Reoptimised the coil positions to have a 3cm clearance from the 100% beam circles rather than a 1cm clearance.
II.C.1, line 4: Insert a comma after "mid-plane".

OK.

II.C.3, LL. 10-11: P-1>> 1 may be true for ISIS and few-GeV proton drivers, but isn't for synchrotrons in general (e.g., LHC).

Rewrote the middle of this section to make it clearer what I was trying to do: "For a synchrotron, $C=P^{-1}$, determined only by the drift space requirements. The original MURA note considered FFAGs entirely filled with magnets ($P=1$) when it quoted values of $C=5$--6 for scaling FFAGs, so CP may be a better metric for comparing lattice bending efficiency independently of packing factor."

III. Table I: Why "Superperiods"? "Periods" or "Periodicity" would be sufficient for single cells.

Changed to "Periodicity: 80 cells per ring"

The column structure, with sometimes one, sometimes two, and sometimes three entries per row, is confusing. Showing the cell borders would make the table a lot clearer.

Added borders to show where the column grouping changes, editors may wish to fine-tune the appearance of this table.

II.C, Fig.9: Shouldn't the vertical axes be labeled simply "Beta (m)"? Isn't the beta function fully determined by the lattice parameters and independent of the beam distribution?

Yes and no. These beta functions were determined from a tracking a beam of the nominal size and using the RMS of moments of that distribution. Due to nonlinear optics, this will differ slightly from the "theory" beta function that considers only infinitesimal deviations from the closed orbit. Caption changed to read "RMS beta functions of the beam at injection in the two lattices, [...]"

III.A, Fig. 12: The large dark green area in the plot doesn't correspond to any box in the Transmission key, though the 100% and 99.9% boxes do contain faint green circles.

Cleaned up the key to only show the range of transmissions attained and replaced the open green circle by a filled one as in the figure.

IV, Figs. 13 & 15: Replace the left-hand axis labels "Thousands" by "kV".

Done.

IV.B, Table III: "ISIS Protons In" and "ISIS Protons Out"; Replace computer notation by scientific notation.

Done.

The ΔQx,y and ΔQv values seem impractically large.

Theory would say they are, but these tune shifts (up to -0.5) are observed on operating high power machines such as ISIS. I've added explanation and two references. The 2.18MW VFFAG has a peak tune shift of -0.395 in the V plane, which does not touch the integer as Qv_zerocurrent=4.433.

V.A, lines 6-8: Rephrase for clarity: e.g. "the diagonal shape between the magnets is makes it difficult to fit in a slotted rf cavity into that spans the whole energy range."

Changed to "the diagonal shape between the magnets makes it difficult to fit in a slotted RF cavity that spans the whole energy range."

V.B, para.1: While it's true that there's no orbit pile-up horizontally in a VFFAG, the orbits do pile up vertically: dy/dE= 1/kE0β2γ. The 1/γ dependence at high energy E0γ is certainly gentler than in a cyclotron, where dr/dE = R/E0βγ3, but the characteristic distance 1/k will also be very much smaller than the "cyclotron radius" R (the radius where β=1).

Rephrased this to say "[isochronism] is easier than in a horizontal orbit excursion FFAG (or a cyclotron) because the orbits do not have to converge towards a single location at the circumference corresponding to $v/c=1$. Such convergence would require magnetic gradients that increase faster than the beam momentum and eventually cause overfocusing, whereas vertical orbit motion can control the field gradient and even keep the normalised focusing strength constant."

Figure 1 suggests that an energy gain of 2 GeV/turn is needed to separate the final turns; 2 GeV/turn in a 12-GeV cyclotron would probably also give adequate turn separation. A numerical comparison between a VFFAG and a cyclotron of the same energy would strengthen the argument.

Adding 2GeV/turn of RF would produce a CW beam power of 34GW in the proton driver ring if it were isochronous and accelerating the same 2.8 Amp (avg.) beam of the pulsed machine; it would also accelerate in only 6 turns. Fortunately such extreme measures are not necessary and a combination of a lower current, smaller size CW beam with less RF is possible – a worked example with numbers is now added to this section.

V.B.1, line 11: Is "normalized focusing" a widely-used term? "normalized focusing strength" (as at the top of p.10) would be clearer.

Fixed.

V.B.2: A reassuring statement about the electrons' energy loss by radiation in the VFFAG arcs would be helpful.

Added "Energy loss from synchrotron radiation is higher than in a comparable ring without reverse magnet bends but is still manageable." A bit more work has actually been done on the removal of synchrotron radiation and the eRHIC VFFAG magnet shape but is not yet ready for publication.

References: In general these do not seem to follow Phys. Rev. style guidelines. The author should consult the Physical Review Style and Notation Guide at:
https://publish.aps.org/files/styleguide-pr.pdf

- Titles are not needed for journal papers

Removed all the titles from the references, makes them a bit opaque but hopefully still usable.

- The names of unfamiliar conference proceedings should be spelt out (e.g. HB2010) and, if possible, page numbers and publication details specified.

Added the full name of HB2010 and page numbers of the publications within conference proceedings.

- Reports should be referred to as "(unpublished)".

Added "(unpublished)" to my own unpublished (web) notes, not sure about other reports.

Although the reports cited in [1] and [3] predate publication of the work in Symon et al.'s 1956 Phys. Rev. article, the latter would be a more generally useful and accessible reference. And although the scaling equation referred to in [1] may have first appeared in that form in Powell's report, it was Symon, Ohkawa and Kolomensky (independently) who had first suggested it.

The 1956 paper is more comprehensive so I've added it as a primary reference but retained the other two as alternatives (so I reference [1,2] or [1,4] in the text).

Style: Phys. Rev. generally follows the AIP Style Manual
http://www.aip.org/pubservs/style.html
where "focusing" is given as the preferred spelling (as in Webster and the OED). Also "rf" and "cw" are the preferred abbreviations for radiofrequency and continuous-wave (using the same case as the original words).

Changed these three throughout. Sometimes "rf" appeared at the beginning of a sentence or in title case so I capitalised it to "Rf".

Referee B Corrections

This paper by S. Brooks is of highest interest to the accelerator community. I found the author's excellent works of the derivation of VFFAG principle followed by the beam simulation using 3-D magnetic field in the VFFAG magnet. As the author mentioned the last sentence of the section II-C-1, the winding configuration can be a quite big issue. I look forward the results in further studies, how it affects focussing properties.

I recommend acceptation. The paper can be published after following minor additions and corrections:

1. The last paragraph in the section II-B, the word 'relativistic', generally, 'ultra relativistic' is used for the case v~c.
Assuming this is section I-B, changed "relativistic" to "ultra relativistic".

2. No explanation for the distance 'a' in Fig. 3 (c).
Added "a" and an arrow to the figure part (c).

3. The arrows in Fig. 3 are not clear.
Made the arrow heads larger and coloured the B field arrows purple similar to the other figures.

4. What kind of code has been used to calculate 3-D magnetic field?
Clarified "The note \cite{VFFAGfields} derives this formula and a Taylor series extrapolation of the field for $x\ne0$. The author wrote a program in C to calculate the field at any point by summing this Taylor series."

5. In Fig. 9, what is the definition of 'RMS beta'?
Defined this in the caption, also added "distribution" to emphasise this came from tracking a finite-sized beam, not linear optics.

6. No explanations for Fig. 11.

7. What kind of tracking code has been used to make Fig. 1, 10, 11?
e.g. Runge-Kutta, symplectic kick and drift, etc.
Added a short paragraph at the end of section III: "Transverse multi-particle tracking without space charge was performed using a 2\nd order drift-kick tracking method, which is symplectic apart from large-angle terms (e.g. spherical aberration) and 200 steps per cell. The results for a 10000 particle distribution are shown in Fig. \ref{multiturn}."

8. What are the criteria to evaluate dynamic aperture of Fig. 12?
e.g. # of turns to track, set physical aperture, etc.
This is explained in section III.A. ("For each square, 250 protons from a 150\,mm.mrad waterbag beam were tracked for 250 cells and removed if $r>10$\,cm.")