NON-SCALING FIXED-FIELD PROTON ACCELERATOR WITH CONSTANT TUNES*

S.J. Brooks†, Brookhaven National Laboratory, Upton, NY, USA

Abstract

Recent studies by Dejan Trbojevic [1] have confirmed that Non-Scaling Fixed Field Accelerators (NS-FFAs) can have their tune dependence on momentum flattened by adding nonlinear components to the magnet fields, although not necessarily for an unlimited momentum range. This paper presents such a cell suitable for the proposed 3–12 MeV FETS-FFA proton R&D ring at RAL [2, 3].

The nonlinear magnetic field components are found automatically using an optimiser and settings covering a ring tune range of one unit in both planes independently are attainable. A fully configurable magnet with multiple windings across its horizontal aperture has been designed in 2D using Poisson, which can produce the required nonlinear fields without exceeding 5 A/mm^2 current density.

INTRODUCTION

The ISIS-II project [4] aims to increase the proton beam power driving the neutron source at RAL to ≥1.25 MW from its present ~200 kW. This is achieved by increasing the injection and extraction energies from the main accelerator to alleviate space charge limits, as well as increasing the repetition rate from 50 to 100 Hz, enabled by using a fixed-field accelerator (FFA) without large ramping magnets.

A proton FFA of this energy and power level would be new technological territory, so a prototype FFA ring at lower energy [2, 3] is being designed to test space charge beam dynamics under machine errors in a realistic setting. This ring is called FETS-FFA as it uses the 3 MeV output proton beam from the Front End Test Stand (FETS) [5, 6] at RAL. Its goal parameters are given in Table 1.

Table 1: FETS-FFA Fixed-Tune Cell Requirements

Parameter	**Value**	**Unit**
Species	Proton	
Kinetic energy	3–12	MeV
Average radius	4	m
Long drift length	1	m
Tune variation	<0.01	per ring
	<0.001	per cell
Tune tunability range	1	per ring
Dynamic aperture	1250	mm.mrad (geom.)

Constant tunes are desirable in high intensity machines to avoid space charge resonance conditions. Thus, the baseline design for FETS-FFA is a scaling type FFA, which guarantees machine tunes stay constant for all energies via the scaling law. Unfortunately scaling machines always have some magnets that are entirely reverse bending, increasing the maximum field requirements for a given machine size. This tension inspired Dejan Trbojevic to search for a variant of the more magnetically efficient non-scaling FFA that also has fixed tunes, initially in the setting of medical proton machines [1]. By carefully adding higher-order multipoles to an initial non-scaling cell design, the sextupoles can cancel the first tune derivative $\frac{\mathrm{d}Q_{x,y}}{\mathrm{d}E}$, the octupoles can cancel $\frac{\mathrm{d}^2Q_{x,y}}{\mathrm{d}E^2}$ and so on. This eventually leads to a very flat tune dependence, at least over a finite energy range. This paper examines such a cell designed for the FETS-FFA parameters.

FIXED-TUNE CELL

A fixed tune cell design for the nominal FETS-FFA tunes is given in Table 2. These elements are aligned around a 4 m radius circle and the FODO cell contains short (OS) and long (OL) drift spaces. The full ring contains 12 such cells (although the baseline in [3] now has 16 cells) for ring tunes of $Q_x^{\mathrm{ring}} = 2.592$ and $Q_y^{\mathrm{ring}} = 2.556$.

Table 2: Lattice Cell for $Q_x = 0.216$, $Q_y = 0.213$

Element	**F**	**OS**	**D**	**OL**	**Unit**
Length	0.5253	0.1	0.4691	1	m
Angle	0.1313	0.025	0.1173	0.25	rad
Fringe σ	0.06		0.06		m
B_0	-0.0169		-0.4034		T
B_1	1.0107		-0.5520		T/m
B_2	-0.9416		3.0345		T/m^2
B_3	-0.9379		2.6260		T/m^3
B_4	-0.5760		-5.0536		T/m^4
B_5	-0.6331		-8.1518		T/m^5

The range of closed orbits is shown in Fig. 1. It can be clearly seen that different energy orbits are dissimilar in shape, so this is not a scaling FFA. Figure 2 shows that the cell tune variation has been controlled to the 10^{-3} level.

The transverse magnetic field profiles of the F and D magnets are plotted in Fig. 3. The full field model uses rectangular magnets (aligned "on average" around their arc segments) with mid-plane field given by:

$$B_y(x, 0, z) = \Phi\left(\frac{z}{\sigma}\right)\Phi\left(\frac{L-z}{\sigma}\right)\sum_{n=0}^{5} B_n x^n$$

in magnet local coordinates. Here, $\Phi(z) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}(z/\sqrt{2})$ is the cumulative probability function of the standard normal

---

* Work supported by Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy.

† sbrooks@bnl.gov

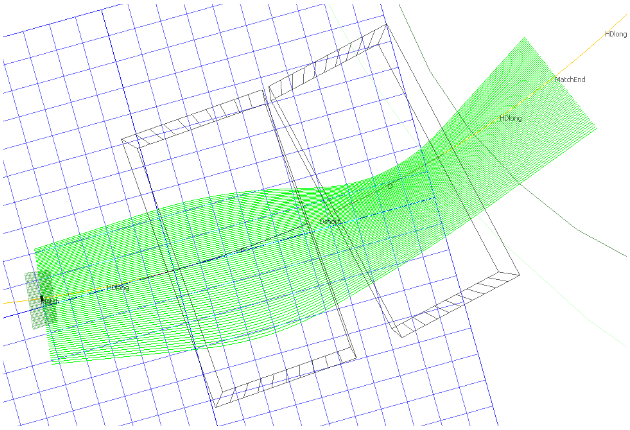

Figure 1: Orbit range in the $Q_{x,y} = (0.216, 0.213)$ cell, from 3–12 MeV (top to bottom). 10 cm grid is shown.

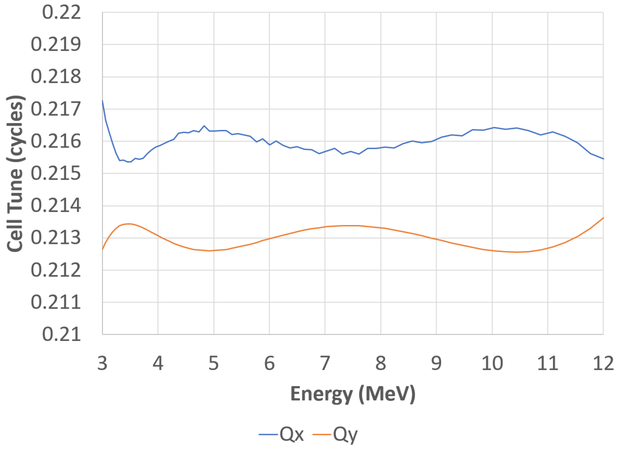

Figure 2: Tune variation in the $Q_{x,y} = (0.216, 0.213)$ cell.

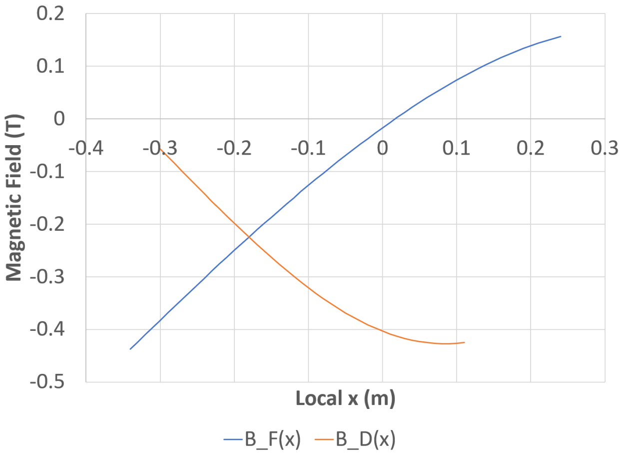

Figure 3: Field profiles for the $Q_{x,y} = (0.216, 0.213)$ cell. Note that higher energy is toward negative x.

distribution, which defines the fringe fields around $z = 0$ and $z = L$. The multipole coefficients B_n, magnet length L and fringe length σ are given for each magnet in Table 2.

The multipole coefficients were determined by computer optimisation using the following six nested iterations:

0. Runge-Kutta 4th order tracking step.
1. **Loop** timesteps to get trajectory in cell.
2. **Finite difference** initial position and angle to get transfer matrix (also gives tunes if orbit is closed).
3. **Optimise** (Newton) to find closed orbit.
4. **Loop** over all FFA energies.
5. **Finite difference** magnet parameters to get response matrix of tune functions to multipole changes.
6. **Optimise** (Levenberg–Marquardt) to make lattice tune functions constant with energy.

An analytic study of a fixed-tune cell has also been attempted [7] but only in the very simple thin-lens, paraxial approximation. It shows that there is one additional free parameter beyond what is allowed in scaling FFAs, while still keeping cell tunes constant. This allows one chosen energy to have equal positive bending fields, for example.

Tune Adjustment Range

Another requirement of the FETS-FFA ring for R&D purposes is that the ring tunes must be adjustable over a full unit, which corresponds to a 0.0833 change in cell tune here. Table 3 shows results from the same algorithm producing fixed-tune lattices for all four “corners” of the $0.2083 \leq Q_{x,y} \leq 0.2917$ tune adjustment range, assuming that all the magnet multipoles can be set independently.

Table 3: Adjusted Tune Designs Summary

Cell Q_x	**Cell Q_y**	**Max tune error**	**Max field (T)**	**Orbit range (m)**
0.216	0.213	0.00126	0.4310	0.573
0.2083	0.2083	0.00074	0.5204	0.641
0.2083	0.2917	0.00185	0.6128	0.688
0.2917	0.2083	0.00168	0.4466	0.389
0.2917	0.2917	0.00037	0.4004	0.381

Dynamic Aperture Test

A brief test of dynamic aperture was performed by launching particles at eight offsets $(\Delta x, \Delta y) = (\pm\delta \text{ or } 0, \pm\delta \text{ or } 0)$ and tracking them for 60 μs. At 3 MeV this is 58 turns (697 cells) and at 12 MeV, 105 turns (1266 cells). The largest value of δ for which all particles survived was 2.5 cm, corresponding to ~400 mm.mrad of geometrical aperture at all energies. This dynamic aperture is less than the 1250 mm.mrad specification, which was achieved after much more optimisation of the tune working point [3], but is still larger than the beam itself.

HORIZONTAL OMNI-MAGNETS

Tune adjustment in FETS-FFA requires independently adjusting all the magnet multipoles: an unusual requirement. A linear version of the round configurable “omni-magnet” described in [8] has been designed to satisfy the large orbit

Table 4: Magnet Performance Summary

Cell Q_x	Cell Q_y	Magnet	Max current density (A/mm^2)	Max B_y error (Gauss)	Max field (T)	Orbit range (m)
0.216	0.213	F	2.372	1.88	0.4224	0.573
		D	4.256	2.32	0.4270	0.400
0.2083	0.2083	F	2.090	2.25	0.5201	0.641
		D	3.111	1.98	0.3921	0.460
0.2083	0.2917	F	3.056	3.31	0.6126	0.688
		D	2.955	2.31	0.4260	0.471
0.2917	0.2083	F	3.073	2.23	0.4418	0.389
		D	3.189	1.89	0.3370	0.247
0.2917	0.2917	F	3.480	2.50	0.3883	0.381
		D	3.944	2.58	0.4035	0.224

excursion of the FFA while preserving this universal field adjustment capability.

Figures 4 and 5 show the same magnet with embedded pole-face coils powered in two different ways, producing the required nonlinear mid-plane fields for two different cases.

Table 4 shows the accuracy of this magnet design for each tune point, as well as the required current densities, which are all less than 5 A/mm^2. Figure 4 is the case with largest peak field and orbit excursion, while Fig. 5 has the smallest orbit excursion and only produces the nonlinear field in a fraction of its horizontal aperture.

Table 5 gives the dimensions of the magnet design.

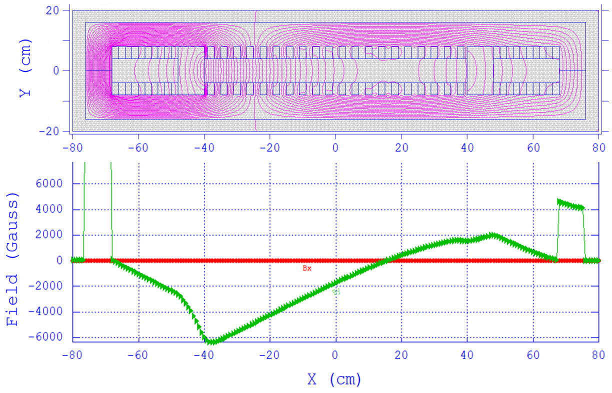

Figure 4: Field lines (top) and mid-plane field (bottom) for the F magnet of the $Q_{x,y} = (0.2083, 0.2917)$ cell.

Table 5: Horizontal Omni-Magnet Geometry

Parameter	Value	Unit
Full aperture	80×8	cm W×H
Main coil size	8×16	cm W×H
Winding size	2×4	cm W×H
Winding pitch	4	cm
Number of windings	20 top, 20 bottom	
Back yoke thickness	8	cm
Full magnet size	152×32	cm W×H

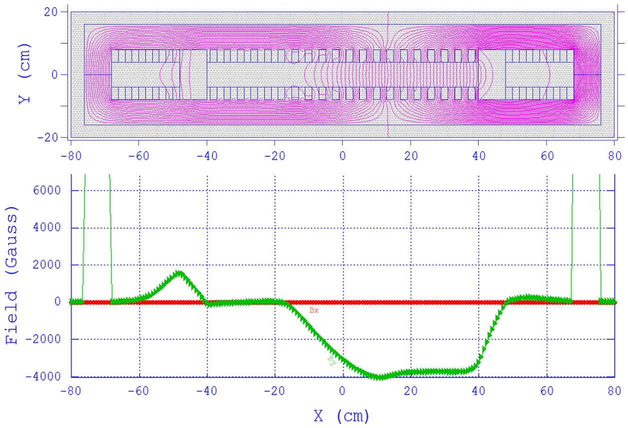

Figure 5: Field lines (top) and mid-plane field (bottom) for the D magnet of the $Q_{x,y} = (0.2917, 0.2917)$ cell.

CONCLUSION & FUTURE WORK

Non-scaling FFA cells with tunes fixed to the $\sim 10^{-3}$ level using nonlinear fields have been found for the parameters of the FETS-FFA project. These were found by an automated process rather than the hand-tuning used in [1]. The dynamic aperture of 400 mm.mrad (geometric) is good by most standards but further improvement will be beneficial here, for example by optimising the tune working point.

The elimination of reverse bends keeps all magnetic fields below 0.62 T on the beams, compared to 1 T or more in the scaling FFA design. This magnetically efficient, fixed tune, non-scaling FFA is a very promising machine type for applications involving many turns and high space charge levels where resonances must be avoided.

Future studies could compare these rectangular magnets to sectors or those with edge angles. The polynomial representation of the transverse field variation also becomes difficult to optimise for high multipole orders; it is suspected that using a Fourier series (or the Chebyshev polynomial basis) would be numerically better conditioned. Some noise in Q_x in Fig. 2 also suggests the accuracy of the tracking code's closed orbit algorithm could be improved.

REFERENCES

[1] D. Trbojevic, S. Brooks, J. S. Berg, and M. Blaskiewicz, "Fast Cycling FFA Permanent Magnet Synchrotron", presented at the 13th Int. Particle Accelerator Conf. (IPAC'22), Bangkok, Thailand, June 2022, paper MOPOST029, pp.126–129, `doi:10.18429/JACoW-IPAC2022-MOPOST029`

[2] J.-B. Lagrange, D. J. Kelliher, A. P. Letchford, S. Machida, C. R. Prior, C. T. Rogers, J. Pasternak, C. Brown, E. Yamakawa, and S. Brooks, "FETS-FFA Ring Study", presented at the 12th Int. Particle Acc. Conf. (IPAC2021), Campinas, SP, Brazil, May 2021, paper TUPAB208, pp.1901–1904, `doi:10.18429/JACoW-IPAC2021-TUPAB208`

[3] S. Machida on behalf of the FFA task team, "Developments and Prospects of FFAs at RAL", presented at the 23rd Int. Conf. on Cyclotrons and their Applications (CYC2022), Beijing, China, December 2022, `https://www.cyc2022.cn/event/1/page/12-programme`

[4] S. Machida, "FFA Options for ISIS Upgrade and the Feasibility Study", Proc. 3rd J-PARC Symposium (J-PARC2019), JPS Conf. Proc. **33**, 011005 (2021), `https://doi.org/10.7566/JPSCP.33.011005`

[5] A. Letchford *et al.*, "Status of the RAL Front End Test Stand", presented at the 4th Int. Particle Accelerator Conf. (IPAC'13), Shanghai, China, May 2013, paper THPWO086, pp.3963–3965.

[6] A. Letchford *et al.*, "Status of the RAL Front End Test Stand", presented at the 6th Int. Particle Accelerator Conf. (IPAC'15), Richmond, VA, USA, May 2015, paper THPF105, pp.3959–3961, `doi:10.18429/JACoW-IPAC2015-THPF105`

[7] S. Brooks, "Paraxial, Thin-Lens Analysis of Fixed-Tune, Non-Scaling FFAs with Two Magnets per Cell", technical report C-A/AP/667 or BNL-223605-2022-TECH, `doi:10.2172/1895077` or `https://www.osti.gov/biblio/1895077` (2022)

[8] S. J. Brooks, "Configurable Field Magnets for a Proton Beam Dynamics R&D Ring", presented at the 4th Int. Particle Accelerator Conf. (IPAC'13), Shanghai, China, May 2013, paper THPME041, pp.3603–3605.